14th World Congress on Computational Mechanics (WCCM)
ECCOMAS Congress 2020
19–24 July 2020, Paris, France

LINEARITY CHARACTERISTICS OF A LOGARITHMIC FINITE ELEMENT BEAM MODEL

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Key Words: Beam theory, Logarithmic finite element method, Multigrid methods

The Logarithmic finite element (LogFE) method extends the Ritz-Galerkin method to approximations on a non-linear finite-dimensional manifold in the infinite-dimensional solution space and allows for a novel treatment of the rotational component of the deformation [1].

We consider a beam discretized by multiple finite elements. The positions and rotations (i.e. the field values) of the nodes fully define the configuration of the beam. Thus, they provide a chart (i.e. a parameterization) on the approximation space, a submanifold of the solution space. The field values at the two ends of the beam depend solely on the coefficients of a single LogFE element modeling the entire beam. The remaining field values depend on both the coefficients of that LogFE element and the coefficients of standard beam elements. The model thus defines an alternative chart on the approximation space, which is diffeomorphic to the chart given by the direct (Cartesian) sum of the field values of the nodes.

Geometrically, the reaction of a beam to a load is generally highly non-linear. However, the degree of non-linearity with regard to the coefficients of a particular beam model depends on the specific parameterization of the approximation space by the coefficients. The model described above parameterizes the approximation space in such a way that the relation between load increments and displacements, in the chart given by the coefficients, is closer to a linear relationship than in standard beam models.

The linearity of a model with regard to a load can be defined with regard to the energy norm that the approximation space inherits from the solution space. We consider a load, $\mathbf{f}(s)$, parameterized by $s \in \mathbb{R}$ such that $\|\partial_s \mathbf{f}\| \equiv 1\,\mathrm{N}$. At a quasi-static equilibrium at position $\mathbf{x}$, an incremental change of the load, $\mathrm{d}\mathbf{f} = \partial_s \mathbf{f}\,\mathrm{d}s$, results in an incremental displacement, $\mathrm{d}\hat{\mathbf{u}} = \partial_s \hat{\mathbf{u}}\,\mathrm{d}s$. Given the stiffness matrix $\mathbf{K}$, we consider the estimated displacement $\bar{\mathbf{u}}$ given by $\mathbf{K}(\mathbf{x} + h\,\partial_s \hat{\mathbf{u}})\,\bar{\mathbf{u}} = \mathbf{f} + h\,\partial_s \mathbf{f}$, $h \in \mathbb{R}$, and its magnitude in the energy norm, $\|\bar{\mathbf{u}}\|$. The degree of nonlinearity is given by the second derivative of $\|\bar{\mathbf{u}}\|$ with regard to h.

In a multigrid setting, the two sets of coefficients do not enter concurrently, but sequentially, into the calculation. We will explore how the increased linearity of the model can lead to a smaller contraction factor C in the inequality $\|\mathbf{e}_{n+1}\| \leq C\,\|\mathbf{e}_n\|^p$, with error $\mathbf{e}_n = \hat{\mathbf{u}}_n - \mathbf{u}$ at iteration n, at least partially offsetting the lower order of convergence p of the optimization algorithm of the multigrid model.

REFERENCES

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