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Analysis of Applications and of Conceptions for an Application-Oriented Mathematics Instruction

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SUMMARY

In connection with the (revived) demand for considering applications in the teaching of mathematics, various schemata or lists of criteria have been developed since the end of the sixties, which set up requirements about closeness to the real world or about the type of mathematics being used, and which have made it possible to analyze the available applications in their light.

After having stated the problem (in section 1), we present (in section 2) a sketch of some of the best known of these and of some earlier schemata, although we are not aiming for a complete picture. Then (in section 3) we distinguish among different dimensions in the analysis of applications. With this as a basis, we develop (in section 4) our own suggestion for categorizing types of applications and conceptions for an application-oriented mathematics instruction. Then (in section 5) we illustrate our schemata by some examples of performed evaluations. Finally (in section 6), we present some preliminary first results of the analysis of teaching conceptions.

1. INTRODUCTION

Since 1975, we (that is, a research team at the University of Kassel) have been involved with questions about an application-oriented mathematics instruction. Early on, one goal of our work became the creation of a *bibliography* and a clear *categorization* of important publications with examples, analyses and general considerations about application-oriented mathematics instruction. In the course of this work it became obvious that *schemas* for *analysing examples of applications*, as well as relevant *instructional conceptions*, were not only helpful but indeed necessary for enabling us,

—to better evaluate publications at a glance with regard to their usefulness
—to index the literature in question clearly and in a unified way, keeping the essential aspects in mind
—to better compare publications with each other and thus to describe and evaluate trends and changes as well as shortcomings
—to point up the discrepancies between the claims of general theory and concrete suggestions
—to better direct the course of our own concrete developmental projects.

Our analyses comprise in the sense just mentioned general considerations and examples on the conference-topic '*teaching of mathematical modelling*', too. For example, our schemas also help us

—to better comprehend different aspects of the topic 'teaching of mathematical modelling', such as the process of modelbuilding, goals, functions of applications or their connection to the real world, and to keep these aspects separate from one another.

With our schemas, we have tried to consider, among other things, the following *aspects*:

—discrimination between the analysis of teaching conceptions and the analysis of examples of applications and therefore (see section 3) discrimination between a 'normative' and a 'descriptive' use of different dimensions.

Here we want to proceed in a *pragmatic* manner; that is, we want to propose *practical* schemas. We do *not* intend to develop either something like a '*theory* of the analysis of applications' or a very *detailed* pattern of categories but

—attainment of a middle way with regard to the degree of precision: the schemas should be both as precise and as practical as possible, both in its dimensions and with reference to their number.

2. SOME CURRENT AND EARLIER SCHEMATA FOR ANALYSING APPLICATIONS

In literature, one firstly can find

(1) *Schemata with the (only) dimension 'nature of connection to the real world'*
Such schemas analyse *single* applications. Here, the schema of Pollak (1969) can be considered as fundamental. Pollak distinguishes between the following types of applications:

—Immediate use of mathematics in everyday life
—Problems that use words from everyday life, from other scholarly or engineering disciplines, to make the problem sound good, but the connection to reality is often false in one or more ways

—Problems that are made to look like applications, which can be described as pure whimsy
—Genuine applications to real life or to other disciplines whose solution requires mathematization
—Puzzles and games which are totally non-practical

(Pollak, 1969, p. 393–404).

It is not very well known, even in Germany, that some schemas for classifying arithmetic problems had already been developed in the early twentieth century, in the framework of discussions about the 'methodology of arithmetic' ('Sachrechnen'). These schemas are partly more precise and more consolidated than proposals today. The fundamental schema, to which all further classifications refer, was published by Kühnel (1916). He distinguishes between the following types of (arithmetic) problems:

—Applied problems, which can be subdivided into problems whose formulation the students are supposed to work out themselves and problems with prescribed formulations
—Word problems, which can be subdivided into problems which are true to life and problems which are remote from real life
—Puzzles and games
—Exercises using weights and measures and others not using weights and measures

(Kühnel, 1916, p. 16–32, 44–90).

A similar schema was developed by Kruckenberg (1935), wherein the connection to the real world is more differentiated. The criterion of differentiation 'life-relevance of the problems for the students' is an *additional* dimension of this schema. The same holds for another schema from the English-speaking countries which was published by Burkhardt (1981). It emphasizes the relevance of the situations to the present and future life of the students and the role of the applications in the process of learning. Thus, these schemas lead up to

(2) *Schemata with several dimensions*
Such schemas analyse applications as well as teaching conceptions—without always clearly distinguishing this—with regard to *different dimensions*. These dimensions are mostly arranged into graduated *categories*.

The most elaborate schema of this type, which is well-known in the German language area, was published by Becker *et al.* (1979). It is an expansion of a schema of Giles (1966). Becker *et al.* propose the following dimensions relevant for evaluating applications:

—nature of the problem's connection to the real world
—source of the data

—number of possible solutions or type of solutions
—difficulty of translating the problem into mathematical language
—elegance of the solution
—scope of the problem
—difficulty of finding appropriate techniques to solve the problem
—complexity of the mathematical techniques
—relevance to other areas
—transferability of the problem-solution or central ideas
—function of the applications within the course
—relation of the applications to the mathematical part of the course
—connections between the various applications
—relation of the applications to other subjects or to other sciences
—connection to the range of experience of the students
—method of presentation of the problem or of the instructional material

(Becker *et al.*, 1979, p. 11–22).

In 1974 Lörcher developed a schema which tries to evaluate applications with regard to their relevance for students. He proposes the following dimensions:

—realm from which the application is drawn and its connection to the real world
—problem-orientation of the treatment of the topics
—transferability of the problem-solution or of central ideas to other situations
—relevance of the results for the students
—relevance of the data for the students

(Lörcher, 1974, p. 76–82).

3. DISCRIMINATION BETWEEN DIFFERENT TYPES OF DIMENSIONS

A closer look at the schemas shows that these proposed dimensions differ from each other in the following ways:

(a) *Conceptional dimensions*: Some dimensions are not related to applications, but only to *teaching conceptions*. Examples are 'significance of applications in instruction' or 'participation by the students'. Here we are only interested in the '*normative*' use of these dimensions in that we are setting up *requirements for teaching*. We call these dimensions '*conceptional dimensions*'.

(b) *Curricular dimensions*: Some dimensions deal with applications (single applications or sets of applications), but they are only practicable in *connection with teaching conceptions*, in which the applications are included. Examples are 'function of applications', 'connections between the various applications' or 'relations of the applications to the mathematical or extra-mathematical part of the course'. These dimensions can be used '*normatively*' as *requirements for applications* within teaching conceptions; if given applications are included in instructional considerations these dimensions can also be used '*descriptively*' for the *classification of applications*. We call these dimensions '*curricular dimensions*'.

(c) *Situational dimensions*: Some dimensions only concern *single applications* and can be used without reference to teaching conceptions. Examples are 'type of connection to reality' or 'kind of use of mathematics'. Such dimensions can on the one hand be used in a '*normative*', on the other hand in a '*descriptive*' manner. We call these dimensions '*situational dimensions*'.

4. PRESENTATION OF A SUGGESTION FOR ANALYSIS-SCHEMATA

4.1. For analysing *conceptions* for an application-oriented mathematics instruction we use *all* types of dimensions described in section 3, in a *normative* manner:

(*A*1) *Dimensions for analysing conceptions for an application-oriented mathematics instruction*

(1a) Which *goals* should mathematics teaching pursue?
(1b) What *relevance* should applications have?
(1c) How should the *students participate* in this?
(2) Which *function* should applications have?
—Should mathematics contribute to the solution of real problems or to the comprehension of real situations?
—Should applications of mathematics contribute to the development of abilities to translate between the real world and mathematics ('translation skills'), to the development of knowledge about applying mathematics ('metaknowledge') resp. to a general view of mathematics?
—Should the applications illustrate or motivate mathematical concepts? Should they help to organize mathematical areas? Should the use of situations promote general goals?
(3a) What *role* should *mathematical theory* play?
(3b) What *role* should the *extra-mathematical context* play?
(4) How should the *connections* between the various *applications* be?
(5a) How should *mathematics* be *used*?
—routine or recipe-like use of mathematical concepts
—intelligent use of mathematical concepts modified according to the situation
—mathematization of situations and development of appropriate concepts, as also application and interpretation of the results to the original situations?

(5b) Should a *distinction* between *model and the real world* be made? If so, how precise should it be?

(5c) If need be, in what way should the *process of modelbuilding* be considered and which steps does it contain? (for example: idealization, mathematization, mathematical reflections, application resp. interpretation).

(6) Which type of *connection to the real world* should the applications have?
—closeness to real world: real assumptions and real problem-setting.
—remote from real world: real assumptions and unreal problem-setting or unreal assumptions and real problem-setting.
—unreal: unreal assumptions and unreal problem-setting.

(7a) From which *extra-mathematical fields* should the applications be taken?
—situations of everyday life.
—situations from the wider environment such as vocation or business.
—situations from the sciences resp. other subjects.

(7b) From which *mathematical fields* should the applications be taken?
—foundations of mathematics/arithmetic/weights and measures.
—algebra/number theory.
—geometry.
—functions/calculus.
—'applied' mathematics (finite mathematics, numerical methods, stochastics).

To illustrate parts of this schema we use three conceptions: (1) The methodics from *Lietzmann* (1916–1952); Lietzmann is a mathematics teacher and educator, who definitely influenced the German discussion from the beginning till the middle of our century. (2) *Glatfeld* (1983), for the lower secondary level. (3) *Jäger* and *Schupp* (1983), who refer to the stochastic instruction in the German 'Hauptschule'.

(*Ad la*): Lietzmann requires not only formal goals as to train the scientific way of thinking, but also practical goals as to train the students to apply mathematics to real life and to recognize mathematical problems in real life. Glatfeld claims as main goal for applied mathematics to qualify the students to mathematize. Furthermore he demands mathematically oriented goals. Schupp formulates general goals, especially the qualification to mathematize.

(*Ad 5a*): Lietzmann emphasizes intelligent use of mathematical concepts, sometimes situations should be mathematized formulated as 'functional thinking'. Glatfeld requires nearly exclusively mathematizations, also Schupp.

(*Ad 5b*): With Lietzmann the discrimination remains implicit and is not detailed. Glatfeld requires a very precise, theoretically ambitious discrimination of model and real world. Schupp claims a precise distinction between model and real world.

(*Ad* 5*c*): Lietzmann states only the approximate steps of the modelbuilding-process: Picking out the mathematical problem from the environment, problem-solving, translating the results into the real world. Glatfeld differentiates by considering the step from the real situation to the idealized situation. Schupp requires additional steps, which embed the problem-solution in mathematical theories.

4.2 For the analysis of teaching conceptions, it is necessary to evaluate the *accompanying applications*, too. Here we use the *curricular* and *situational* dimensions, in a *descriptive* manner.

(*A*2) *Dimensions for analysing applications included in teaching conceptions*
(1) Which *function* do the applications have? (categories as in A1).
(2a) What *role* does the *mathematical theory* play?
(2b) What *role* does the *extra-mathematical context* play?
(3) What are the *connections* between the various *applications*?
(4a) How is *mathematics* actually *used*? (categories as in A1).
(4b) Is a *distinction* between *model and the real world* made? If so, how precise is it?
(4c) If need be, in what way is the *process of modelbuilding* taken into account and which steps does it contain?
(5) Which type of *connection to the real world* do the applications have? (categories as in A1).
(6a) To which *extra-mathematical fields* do the applications belong? (categories as in A1).
(6b) To which *mathematical fields* do the applications belong? (categories as in A1).

To illustrate parts of this schema, we use the same conceptions:

(*Ad* 4*a*): Lietzmann's applications use mathematics mostly routinely or intelligently, but there are some examples with parts of mathematizations. With Glatfeld one can see a difference from his theoretical requirements. The examples use nearly exclusively mathematics routinely or recipe-like, mathematizations are very seldom. This is different with Schupp, who develops, as theoretically required, mathematizations besides examples with an intelligent use of mathematics.

(*Ad* 4*b*): Lietzmann's applications don't discriminate in this way; which Lietzmann indeed does not intend. Within Glatfeld's examples there exist only a few applications which are based on this discrimination. Schupp, in the contrary, explicates this distinction in his examples very often and carefully.

(*Ad* 4*c*): This tendency is continued with the consideration of the modelbuilding process.

4.3. For analysing *single applications*, not (necessarily) included in teaching conceptions, we firstly use the *situational* dimensions. Among the other dimensions, the dimension '*function* of an application' is also applicable to single applications. It makes sense to base the evaluation upon this dimension too, because one can mostly recognize which function a single application can perform or not perform, e.g. from the context or the type of presentation.

(B) Dimensions for analysing single applications

(1) Which (possible) *function* has the application? (categories as in A1).

(2a) How is *mathematics used*? (categories as in A1).

(2b) Is a *distinction* between *model and reality* made? If so, how precise is it?

(2c) If need be, in what way is the *process of modelbuilding* taken into account and what steps does it contain?

(3) Which type of *connection to the real world* does the application have? (categories as in A1).

(4a) To which *extra-mathematical fields* does the application belong? (categories as in A1).

(4b) To which *mathematical fields* does the application belong? (categories as in A1).

5. SELECTED EXAMPLES OF PERFORMED EVALUATIONS

5.1. First we illustrate *schema A* by the complete evaluation of Lietzmann. (See p. 210/211).

Up to now about 40 teaching conceptions and accompanying applications from the German, English and French language area have been analysed in this way.

5.2. In the framework of a *bibliography* of literature for an application-oriented mathematics instruction (Kaiser, Blum, Schober, 1982), *schema B* was used with slightly modified categories, because the emphasis lay on a systematical analysis of 'relevant' single applications whereas, with regard to teaching conceptions, only a rough survey was intended.

The following dimensions were used:

—type of use of mathematics (in the following example under 2): (categories as in A1).

—type of connection to the real world (in the following example under 3):
  —situations which are close to real world with data which are close to real world
  —'intentional' unreal situations

—function of the application (in the example under 4): (categories as in A1)

—extra-mathematical fields:

—arts	—everyday life	—military science	—sociology
—biology	—geography	—music	—sports

—chemistry —linguistics —political science —technology
—economics —medicine —psychology

—mathematical fields:
—algebra/number theory —functions
—arithmetic/weights and measures —geometry
—calculus —linear algebra/analytic geometry
—finite mathematics —numerical methods
—foundations of mathematics —stochastics

The two last-mentioned dimensions were differentiated with 75 headings in all, to which *indexes* were produced to make a systematic research of literature possible. The bibliography contains 350 selected publications, 235 of these are reported in detail.

Here is an *example* with extracts from the text and the indexes:
Example of a summary:

HENN, H. W.
Preisindices in der Schule.
In: Didaktik der Mathematik, 8(1980a)3, S. 222–238.

ZIEL
Mathematische Analyse und Hinweise zur unterrichtlichen Behandlung von Preisindices.

INHALTE
1. Ausgangspunkt: Verwendung von Preisindizes in tarifpolitischen Auseinandersetzungen und öffentlichen Diskussionen als scheinbar objektive Bestandsaufnahme wirtschaftlicher Verhältnisse.
   Mathematische Präzisierung der Preisindizes: Darstellung der gebräuchlichen Preisindizes, mathematische Forderungen an einen Preisindex, Entwicklung verschiedener mittlerer Preissteigerungsraten wie Inflationsrate, lokale Preissteigerungsrate, spezielle Preisindexfunktionen.
   Vorschläge zur unterrichtlichen Umsetzung:
   —Sekundarstufe I: Zunächst Preisindex für Einzelwaren, dann Preisindex für Warenkorb, Vergleich verschiedener Warenkörbe, Inflationsrate, graphische Darstellung über Jahrzehnte.
   —Sekundarstufe II: Präzisierung, Behandlung weiterer mittlerer Preissteigerungsraten und der lokalen prozentualen Preissteigerungsrate.
   Diskussion der mathematischen Verfahren ausgehend von Tagesproblemen; ständige Rückübersetzung aus der mathematischen Beschreibung in die Realität.
2. Verständige Anwendung mathematischer Verfahren.
3. Realitätsnahe Situationen mit realitätsnahen Daten.
4. Mathematik soll zum Verständnis der Situation beitragen.

WERTUNG
Darstellung mathematischer Zusammenhänge mit unterrichtlichen Hinweisen zur Behandlung aktueller, relevanter Anwendungen und zur Vermittlung grundlegender Übersetzungsqualifikationen. Unterrichtsvorschlag für die Sekundarstufe I direkt umsetzbar, beim Unterrichtsvorschlag für die Sekundarstufe II sind method. Überlegungen nötig. Kenntnis wirtschaftl. Zusammenhänge hilfreich. Unterrichtliche Verwendbarkeit: In fächerübergreifenden Unterrichtseinheiten ab Jg. 7, bei Behandlung von Folgen in Jg. 10, in Analysiskursen ab Jg. 11/II.

	Analysis of Lietzmann's teaching conception	Analysis of Lietzmann's applications
Goals of mathematics teaching	Formal goals should be to train the scientific way of thinking, to become familiar with weights and measures, and to develop an intuitive faculty. Practical goals should be to train the students to apply mathematics to reality and to recognize mathematical problems in reality. Furthermore aesthetic and ethical goals such as education to objectivity.	
Relevance of applications	In the 'traditional practical mathematics' (Sachrechnen), especially in the last year of the 'low level' (Hauptschule), there should be high relevance of the applications by organizing instruction in extra-mathematically structured teaching units; besides applications should be considered by linking the different school subjects.	
Participation	Independent work should be considered in homework, in projects e.g. during stays in country boarding schools.	
Functions of applications	Applications should illustrate mathematical concepts, promote general goals and help to organize mathematical areas; besides, mathematics should contribute to the solution of real problems and to the development of translation skills.	Applications illustrate mathematical concepts and organize mathematical areas; furthermore, mathematics can contribute to the solution of real problems.
Role of math. theory	Emphasis should be on mathematics, even in the 'traditional practical mathematics'.	Explicit, broad treatment in mathematically structured applications.
Role extra-math. context	As a rule, applications should be avoided in which explaining the context takes longer than the arithmetical operations.	Explicit, broad treatment, especially in extra-mathematically structured applications.

Connections	Arrangement in extra-mathematically structured teaching units.	Co-ordination with other sciences and school subjects.
Type of use	Placing the emphasis on intelligent use by avoiding schemas, sometimes situations should be mathematized, formulated as 'functional thinking'.	Frequent routine use of mathematical methods, partly mathematizations with reference to necessary simplifications.
Model and real world	The discrimination between model and real world remains implicit, within the practical goals, especially with reference to necessary simplifications.	No discrimination is made.
Process of model-building	Only few steps should be distinguished: picking out the mathematical problem from the environment, problem-solving, translating the results in real world.	The process of model-building is not considered.
Connection to real world	Emphasis should be on problems which are close to real world, besides imaginative problems also have a right to be included (so-called recreational mathematics).	A three-part division: the largest consisting of applications which are close to real world, a smaller part are remote from real world (mostly because of imaginary problem-setting) and a third part are unreal problems.
Extra-math. fields	Applications should be taken from everyday life, from the wider environment and other school subjects; overly specific relations to vocation and business should be avoided.	Applications belong to everyday life, to the wider environment and to other school subjects.
Mathemat. fields	Applications should be taken from foundations/arithmetic/weights, measures; algebra; geometry; functions/calculus; applied mathematics.	Applications belong to foundations/arithmetic/weights, measures; algebra; geometry; functions/calculus; applied mathematics.

Extract from the indexes:

Volkswirtschaftslehre

BAUMANN 1977b	HENN 1980a	LIETZMANN 1943
BAUMANN 1977c	*HERGET	LING
BENDER, E. A.	HESSISCHES INSTITUT ...	SCHNEIDER 1968
BOSCH; WITTMANN	INEICHEN	*STONE
....	....	....

Differentialrechnung

....	....	....
DREETZ	*MONTGOMERY	*STOCK
*DRENCKHAHN; SCHNEIDER	*NOACK	THEODORE
FREUDENTHAL 1973	NOBLE 1967	*TIETZE
HENN 1980a	OPEN UNIVERSITY COURSE TEAM	TIMISCHL 1980
HENN 1980b	OTTE; STEINBRING; STOWASSER	TWERSKY
*HERGET	*RIEBESELL	*WEYGANG 1976
HOGBEN	DE SAPIO	WODE

Folgen

BOARD OF EDUCATION ...	LIETH	SCHNEIDER 1968
DEUTSCHES INSTITUT ...	LIETZMANN 1924	SCHOOLS COUNCIL SIXTH FORM ...
ENGEL 1968	LING	*SLOYER
HENN 1980a	MENNINGER	STREHL
*HIRSTEIN	MEYER	TANNERT
JACOBS	OPEN UNIVERSITY COURSE TEAM	THEODORE
*KAISER u.a.	ROHRBERG	*WEYGANG 1976
....	....	....

6. SOME PRELIMINARY FIRST RESULTS OF THE ANALYSIS OF TEACHING CONCEPTIONS

Finally, we present some results—still preliminary ones—which G. Kaiser has attained through a detailed analysis of familiar *German* conceptions for an application-oriented mathematics instruction (further analyses show that there are relevant differences between the English and the German language area; we have to omit details here).

At the beginning of this century very precise differentiations were developed according to the connection of the applications to the real world. At that time no distinction was made, however, between model and the real world. Around 1930 there was a vague talk about 'schema' or 'model'. Only after World War II do we find the beginning of a distinction between model and real world, and a development of ideas resembling today's conceptions of modelbuilding. But all this remained rather imprecise. Only *in the seventies*, in connection with a strong trend toward applications in mathematics instruction, *precise notions* about the *distinction between model and real world* and about *possible procedures in the mathematizing process* were developed. These notions, according to our analysis, have met with *general agreement* since the end of the seventies in the discussion of an application-oriented mathematics instruction. This consensus has been relatively independent of the different tendencies within application-oriented mathematics instruction. Some writers set up a great variety of ambitious goals, but their suggestions about teaching have little to do with these goals. *Gaps between claims and realization* also appear on other levels, such as the connection of the applications to the real world, the function of the applications, and the proportion of extra-mathematical context to mathematical theory. But these gaps are not as large as those in the realm of *modelbuilding activities*. So, for example, at the present time more modest demands are being made for the connection of the applications to the real world. In recent years, however, increasing numbers of instructional suggestions have been developed which indeed bring in the real world, but which are not based on mathematizations.

After these rather general remarks, we want to add some preliminary results of the analysis, which are more closely related to the process of modelbuilding and to the relation between mathematics and the real world. It became clear that the *demand for realistic examples* on the one hand and for *considerations of modelbuilding* on the other are only *distantly related* to each other. The process of modelbuilding can be carried out using examples unrelated to the real world. In such cases the phases of idealization of the real situation and the interpretation of the results in the initial problem are missing, since the real situation had already been radically simplified. It seems questionable whether any genuine mathematization abilities or metaknowledge about applying mathematics can be conveyed using such abbreviated examples.

Furthermore, the *degree of preciseness* of the *distinction between model and real world* has *consequences for the process of modelbuilding*. So, for example, when we start with situations which have already been idealized, the resulting mathematization appears almost compulsory, which is practically a falsification of any genuine process of modelbuilding.

We are well aware that some of the results just presented may already seem familiar. As we said at the outset, the function and purpose of our analysis-schema is—among other things—to provide a more precise and more carefully differentiated grasp of the gaps between claim and realization, as well as of shortcomings, trends, changes, etc. Thus, already supposed statements can be supported more precisely or, if necessary, can be refuted.

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