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Modelling stress-state dependent nonlocal damage and failure of ductile metals

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Damage and failure of ductile metals is highly influenced by the stress state. In fact, void growth takes place under hydrostatic tensile loading and void coalescence leads eventually to ductile failure. A completely different damage mechanism occurs for shear dominated loading that results in void elongation within a narrow shear band. In order to account for both failure mechanisms, a stress state dependent damage model is proposed. The model is based on a continuum damage mechanics approach, whereby nonlinear damage evolution is taken into account. The influence of the stress triaxiality and the LODE parameter on damage initiation and failure is considered by the Hosford-Coulomb model. Pathological mesh sensitivity is prevented by an integral type nonlocal formulation. Finally, the proposed stress-state dependent nonlocal damage model is verified by test data for the microalloyed high strength steel HX340LAD for a wide range of stress states.

1 Constitutive model

The proposed constitutive model includes a plasticity theory, which is coupled with stress-state dependent nonlocal damage in order to represent the material characteristics of ductile metals. It is assumed that plasticity and damage are isotropic. The VON MISES yield criterion $F = 0$ is employed, whereby the VON MISES equivalent stress $\sigma_{\mathrm{eq}} = \sqrt{3J_2}$ is compared to the sum of the initial yield stress κ_0 and nonlinear isotropic hardening κ.

$$F = \sqrt{3J_2} - (\kappa_0 + \kappa) = 0\,, \quad \kappa = \kappa^\infty \left[1 - \exp\left(-\frac{E_\kappa}{\kappa^\infty}\bar{\mathrm{E}}_{\mathrm{pl}}\right) + \alpha_\kappa \left(\frac{E_\kappa}{\kappa^\infty}\bar{\mathrm{E}}_{\mathrm{pl}}\right)^{m_\kappa}\right] \tag{1}$$

The hardening κ is driven by the equivalent plastic strain $\bar{\mathrm{E}}_{\mathrm{pl}} = \int_0^{t^*} \sqrt{2/3\,\dot{\mathbf{E}}_{\mathrm{pl}} \cdot \dot{\mathbf{E}}_{\mathrm{pl}}}\,\mathrm{d}t$ according to (1b), which contains the four parameters κ^∞, E_κ, α_κ, and m_κ in the hardening function (1b). Since the second deviatoric stress invariant J_2 is applied here only, the yield condition is independent of the hydrostatic pressure. The plasticity model is implemented into the finite element software LS-DYNA within a hypoelastic framework considering finite deformations. The CAUCHY stress tensor $\mathbf{T}$ is coupled with the nonlocal damage variable $\tilde{D}$ by means of the concept of effective stresses, with the nominal CAUCHY stress tensor in the damaged configuration $\mathbf{T}^{\mathrm{d}}$. By applying the integral nonlocal approach according to [1], the nonlocal damage rate $\dot{\tilde{D}}(\mathbf{x})$ at the reference point $\mathbf{x}$ is obtained by weighting the local damage rate $\dot{D}(\mathbf{y})$ at neighbouring points $\mathbf{y}$ over a certain region Ω. Therefore, the weighting function $w(\mathbf{x},\mathbf{y})$ is employed with the two model parameters p and q as well as the characteristic length l_{c}. Hereby, $\|\mathbf{x}-\mathbf{y}\|$ is the EUCLIDean norm of the distance between $\mathbf{x}$ and $\mathbf{y}$.

$$\mathbf{T}^{\mathrm{d}} = (1-\tilde{D})\,\mathbf{T}\,, \quad \dot{\tilde{D}}(\mathbf{x}) = \frac{\int_\Omega \dot{D}(\mathbf{y})\,w(\mathbf{x},\mathbf{y})\,\mathrm{d}\mathbf{y}}{\int_\Omega w(\mathbf{x},\mathbf{y})\,\mathrm{d}\mathbf{y}}\,, \quad w(\mathbf{x},\mathbf{y}) = \left[1 + \left(\frac{\|\mathbf{x}-\mathbf{y}\|}{l_{\mathrm{c}}}\right)^p\right]^{-q} \tag{2}$$

In order to achieve a stress-state dependent nonlocal damage model, the stress triaxiality T is applied by taking the normalised influence of the mean stress σ_{m} into account. Moreover, the LODE angle θ_{L} is introduced by considering the third deviatoric stress invariant J_3. The LODE angle is normalised to the LODE parameter $\bar{L}$, which is defined in the range of $-1 \le \bar{L} \le 1$.

$$T = \frac{\sigma_{\mathrm{m}}}{\sigma_{\mathrm{eq}}}\,, \quad \theta_{\mathrm{L}} = \frac{1}{3}\arccos\left(\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right)\,, \quad \bar{L} = 1 - \frac{6}{\pi}\vartheta = 1 - \frac{2}{\pi}\arccos\left(\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right) \tag{3}$$

Nonlinear damage growth is modelled by means of an evolution equation for the damage rate according to [2], where the equivalent plastic strain based damage initiation criterion $\varepsilon_{\mathrm{c}}(T,\bar{L})$ as well as the failure criterion $\varepsilon_{\mathrm{f}}(T,\bar{L})$ are defined in (4b). Both criteria are functions of the stress triaxiality and the LODE parameter in general. The damage exponent n_{D} controls the nonlinearity of the damage evolution behaviour. Considering a stress-state dependent damage and failure criterion, the HOSFORD-COULOMB model, proposed in [3], is applied for the functions $\varepsilon_{\mathrm{c}}(T,\bar{L})$ and $\varepsilon_{\mathrm{f}}(T,\bar{L})$.

$$\dot{D} = \frac{n_{\mathrm{D}}}{\varepsilon_{\mathrm{f}}(T,\bar{L}) - \varepsilon_{\mathrm{c}}(T,\bar{L})}\left\langle\frac{\bar{\mathrm{E}}_{\mathrm{pl}} - \varepsilon_{\mathrm{c}}(T,\bar{L})}{\varepsilon_{\mathrm{f}}(T,\bar{L}) - \varepsilon_{\mathrm{c}}(T,\bar{L})}\right\rangle^{n_{\mathrm{D}}-1}\dot{\bar{\mathrm{E}}}_{\mathrm{pl}}\,, \quad \varepsilon_{\mathrm{c/f}}(T,\bar{L}) = b_{\mathrm{c/f}}\left(\frac{1+c_{\mathrm{c/f}}}{g_{\mathrm{c/f}}(T,\bar{L})}\right)^{\frac{1}{n}} \tag{4}$$

The triaxiality and LODE parameter dependence on the critical strain ε_{c} and the failure strain ε_{f} is introduced by the functions:

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$$g_{c/f}(T,\bar{L}) = \left[\frac{1}{2}(f_1-f_2)^{a_{c/f}} + \frac{1}{2}(f_2-f_3)^{a_{c/f}} + \frac{1}{2}(f_1-f_3)^{a_{c/f}}\right]^{\frac{1}{a_{c/f}}} + c_{c/f}\,(2T + f_1 + f_3) \tag{5}$$

$$f_1 = \frac{2}{3}\cos\left[\frac{\pi}{6}(1-\bar{L})\right], \quad f_2 = \frac{2}{3}\cos\left[\frac{\pi}{6}(3+\bar{L})\right], \quad f_3 = -\frac{2}{3}\cos\left[\frac{\pi}{6}(1+\bar{L})\right] \tag{6}$$

The HOSFORD-COULOMB approach can be interpreted as a generalised MOHR-COULOMB model with the parameters a_c, b_c, c_c for damage initiation and a_f, b_f, c_f for failure. The exponent in (4b) is defined a priori to $n = 0.1$.

2 Parameter identification and verification for a wide range of stress states

A stepwise procedure is pursued where the parameters for the ductile steel HX340LAD are identified on the basis of the test data carried out by the Fraunhofer Institute for Mechanics of Materials (IWM), see [4]. At first, the plasticity model is calibrated to tensile test data. Then, the parameters of the nonlocal weighting function are estimated by means of the plastic strain distribution prior to failure. Finally, the HOSFORD-COULOMB model parameters are determined. Therefore, the design of experiments includes flat smooth and notched tensile specimens, various shear specimens as well as a NAKAZIMA biaxial punch test in order to capture a wide range of stress states within the diagram of the triaxiality versus the LODE parameter, see Fig. 1 a). The identified damage initiation and failure surfaces are depicted in Fig. 1 b) and c). The simulation results, obtained by the identified model parameter set in Tab. 1, are compared to test data and shown in Fig. 1 d) and e) for the smooth tensile specimen and the 0° shear specimen. A good accordance is achieved regarding the yield stress, the hardening and the damage and failure behaviour leading to a successful verification of the proposed model for a wide range of stress states.

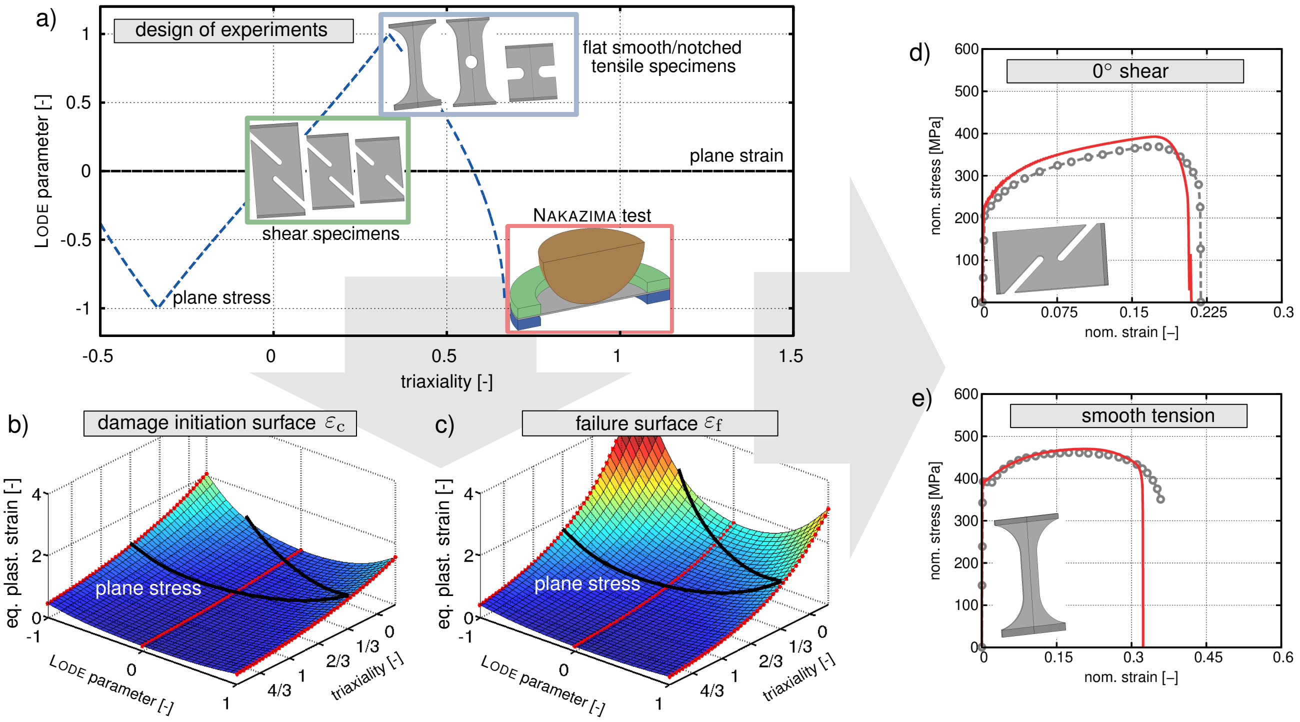

Fig. 1: Design of experiments a), damage initiation surface b), failure surface c) and simulation results compared to test data for smooth tensile specimen and 0° shear test for HX340LAD steel d) and e)

Table 1: Model parameters for the HX340LAD microalloyed steel

E [MPa]	ν [-]	κ_0 [MPa]	κ^∞ [MPa]	E_κ [MPa]	α_κ [-]	m_κ [-]	n_D [-]	l_c [-]
210000.0	0.3	383.69	215.37	1450.59	0.1	1.2	2.0	2.0
p [-]	q [-]	n [-]	a_c [-]	b_c [-]	c_c [-]	a_f [-]	b_f [-]	c_f [-]
2.0	2.0	0.1	1.166	0.9566	0.04433	1.182	1.398	0.07605

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