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The Logarithmic Finite Element Method: Approximation on a Manifold in the Configuration Space

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ABSTRACT

The Logarithmic finite element method extends the Ritz-Galerkin method to approximations on a non-linear finite-dimensional manifold in the infinite-dimensional solution space. Formulating the interpolant on the logarithmic space allows for a novel treatment of the rotational component of the deformation, making this approach especially suitable for geometrically exact formulations involving large rotations. Using homogeneous coordinates, the logarithms of transformation matrices representing rotations and translations are given as elements of a linear subspace of the set of affine transformations, generating a Lie algebra. The degrees of freedom present in a finite element based on the LogFE method are associated with vector-valued shape functions which constitute the basis vectors of that subspace. Given an appropriate formulation of the finite elements, local degrees of freedom related to rotations and translations can be linked to global degrees of freedom and boundary conditions, and the interpolant is given by an immersion of the space of degrees of freedom into the configuration space. Thus, the LogFE method satisfies the general criteria for finite element models as given by Ciarlet [1]. A co-rotational formulation enables the model to exactly represent pure rigid body motions. This co-rotational formulation must also ensure that spurious high-order deformation components vanish with mesh refinement, in order to satisfy the interpolation theorem for finite elements [2]. Expanding on the work in [3], the authors will present co-rotational, geometrically exact formulations for both planar and spatial beam elements endowed with Bernoulli and Timoshenko kinematics. [1] Ciarlet, P.G. (1979) The Finite Element Method for Elliptic Problems. North-Holland, Amsterdam. [2] Oden, J.T. and Reddy, J.N. (2011) An Introduction to the Mathematical Theory of Finite Elements. Dover, Mineola. [3] Schröppel, C. and Wackerfuß, J. (2016) Introducing the Logarithmic finite element method: a geometrically exact planar Bernoulli beam element. Advanced Modeling and Simulation in Engineering Sciences, 3 (1).