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RELATIONS BETWEEN MATHEMATICS AND EMPLOYMENT IN MATHEMATICS EDUCATION IN FULL TIME TECHNICAL AND VOCATIONAL EDUCATION

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1. Some Remarks on the West German System of Full Time Technical and Vocational Education

It would be much too complicated to explain in detail the organisation of the German full time Technical and Vocational Education (TVE) system. There are schools at the lower secondary level (pupils aged about 15 to 16 years) as well as colleges at the upper secondary level (students aged about 17 to 20 years) and colleges for further education (students aged over 20 years). For details about the German TVE system and mathematics education in it, see (8) and (3). All types of these full time schools and colleges provide the student with both a general certificate and a preparation for some vocational area. In all types of schools and colleges mathematics is an important subject. The contents of mathematics education are - - with some exceptions - - roughly comparable to the contents in those schools and colleges in the "general" education system that provide students with equivalent certificates. (In this paper, I cannot consider meaning of or problems in "general" education and mathematics education in it; see (8) and (3).) That means: algebra (equations and functions) and geometry in the lower secondary level, and in the upper secondary level mainly calculus, in addition to analytic geometry and some stochastics.

2. Important Goals of Mathematics Education in Full Time Technical and Vocations Education

I deal neither with "official" nor with "actual" goals of mathematics education in full time TVE. Instead, I propose three goals which in my opinion are all important for full time TVE. (The goals - - explicit or implicit - -actually aspired to in the classroom are very often the "vocational part" of goal 1 or goal 2 and very seldom both together. Roughly speaking, mathematics education in full time technical and vocational schools and colleges is often either an auxiliary subject for technology or business or a self-supporting subject oriented toward the "gymnasium".)

Goal 1:

Mathematics education should contribute to better describing, better understanding, and mastering present and not-too-distant future problems from the student's vocational and everyday environment. Goal 1 has a "vocational" and a "non-vocational" part. Taking into account the latter, preparation for vocations in one but not the only direct use that should be made of mathematics.

Moreover, more important than dealing with particular problems and particular mathematical topics is that the student learn by a few examples to translate between mathematics and the real world, and especially that he or she learns to mathematize meaningful contexts (7) and learn general strategies for tackling real problems, according to Figure 1 (see also 6, 11, 10, 5, 2).

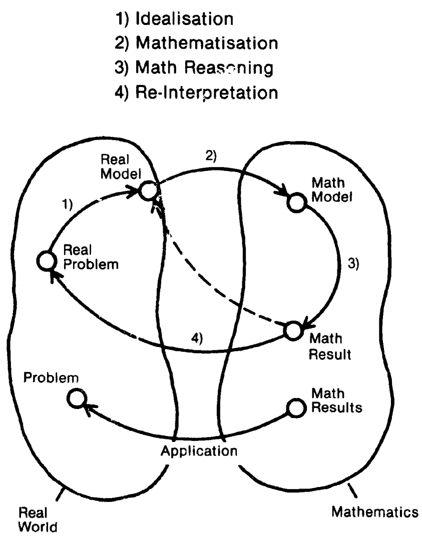

Figure 1

Goal 2:

Mathematics education should try to attain so called formal aims, that is (in accordance with Wittmann (13)): to advance cognitive strategies and intellectual techniques, such as mathematical reasoning and algorithmic thinking.

Goal 3:

Mathematics education should contribute to equalizing the opportunities of students by making possible further learning and advancement to higher education.

3. Implications for the Interface Between Mathematics and Employment in Full Time Technical and Vocational Education

The interface between mathematics and employment in mathematics education works mostly by dealing with situations and problems coming from the corresponding vocational area and containing or leading to suitable mathematical topics. It stands to reason that vocational situations and problems provide important hinds for the mathematics curriculum in order to achieve the "vocational part" of goal 1 (role a).

But, according to goal 1, vocational problems are not the only important ones in full time TVE. As, for example, Strasser (12) points out, the mathematical needs for various jobs and vocations are not as clearly defined as would be desirable. Therefore, vocational situations and problems cannot immediately or even exclusively determine the mathematics curriculum. Instead, such situations and problems (for criteria for suitable application problems, see 9, 10, or 2) play a methodological role as they show by example the interface between mathematics and real world situations (role b), motivate students to study mathematical topics that may have another justification, according to goals 2 and 3 (role c), and illustrate mathematical topics and thus help the student understand them better and retain them longer (role d).

4. An Example: The West German Income Tax Function

As an example of an application that can play all for roles identified in section 3, we look at the German income tax. This example yields a vocational problem from the economic and financial area. At the same time, and perhaps didactically even more important, it yields an everyday problem for all present or future taxpayers. I have had much instructional experience using this example with 17- to 19-year-old students (see also chapter 7 in textbook 1).

4.1 Information

The present West German income tax law runs as follows:

. 32a
Einkommensteuertarif

(1) Die tarifliche Einkommensteuer bemisst sich nach dem zu versteuernden Einkommen. Sie betragt vorbehaltlich der .. 32b, 34 un 34b jeweils in Deutsche Mark

1. fur zu versteuernde Einkommen bis 3719 Deutsche Mark: 0;

2. fur zu verteuernde Einkommen von 3720 Deutsche Mark bis 16019 Deutsche Mark: 0,22x - 812;

3. fur zu verteuernde Einkommen von 16020 Deutsche Mark bis 47999 Deutsche Mark: (((10,86y - 154,42)y + 929)y = 2200)y + 2708;

4. fur zu versteuernde Einkommen von 48000 Deutsche Mark bis 130019 Deutsche Mark: 109,95)z + 4800)z + 15298;

Deutsche Mark an:

"x" ist das abgerundete zu verteuernde Einkommen. "y" ist ein Zehntausendstel des 16000 Deutsche Mark ubersteigenden Teils des abgerundeten zu versteuernden Einkommens. "z" ist ein Aehntausendstel des 48000 Deutsche Mark ubersteigenden Teils des abgerundeten zu versteuernden Einkommens.

Let, as in this law, x be the annual income (in DM), and let s(x) be the income tax in DM) that has to be paid by an unmarried peson earning x. Idealizing, we allow x to be any element of IR . Disregarding rounding off, the German income tax function $s : x \to s(x);\ x \in R^+$ is a piecewise polynomial function. The graph f this is shown in Figure 2.

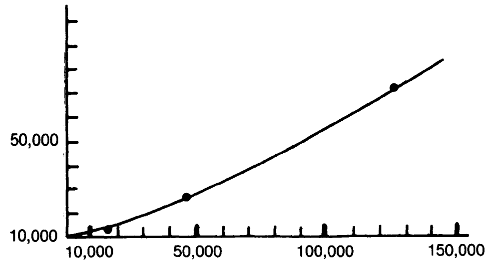

Figure 2

Let $d(x) = s(x)/x$ be the average tax rate for income $x \in R^+$ and $d : x \to d(x);\ x \in R^+$ be the average tax rate function. If, as in the German law, the income tax function is almost everywhere differentiable and everywhere differentiable from the right, let s'(x) be the (right) derivative at $x \in R_0^+$, that is, the instantaneous rate of change of s at x, or the marginal tax rate. Let $s' : x \to s'(x);\ x \in R_0^+$ be the marginal tax rate function. The two functions d and s' are more important in political and public discussions that the function s itself. The graphs of d and s' for Germany are shown in Figures 3 and 4 respectively.

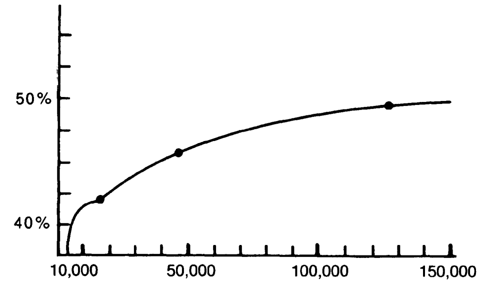

Figure 3

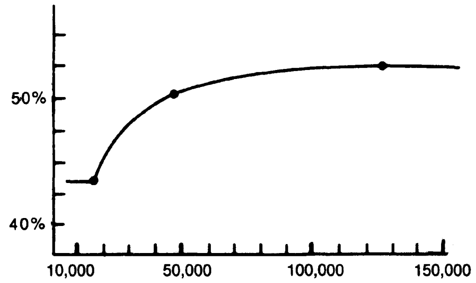

Figure 4

4.2 Income Tax as a Subject in Mathematics Education in Full Time Technical and Vocational Education

Firstly, this example can show - - according to role a in section 3 - - how mathematical topics (like functions and their properties, secants, tangents, derivatives, etc.) can contribute to achieving goal I (with regard to some economical problems). For example, mathematics education can help students understand the idea of tax rates. This is important not only for future tax consultants but for all citizens. Therefore, this example is important not only for vocational colleges in the business area but also for all other types of colleges, where it plays an analogous role to a, according to the "non-vocational" part of goal I. The appropriate mathematical topics (like derivatives) need not necessarily be introduced in a formal and rigorous way. The aim is that students may be able to use these topics intelligently as a tool. Simplifications which are not falsifications are allowed or even necessary (see 4).

Secondly, this example can demonstrate - - according to role b - - the interface between mathematics and real world situations shown in Figure 1 (see 2).

Thirdly, this example yields - - according to roles c and d - - motivations and illustrations for a great many mathematical topics, especially for the uppe secondary level. For example, it is easy to show the importance of determining by means of calculus the regions of increase and of convexity of the polynomial functions that compose the German income tax function (see again 2).

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