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THEME GROUP 6: MATHEMATICS AND OTHER SUBJECTS

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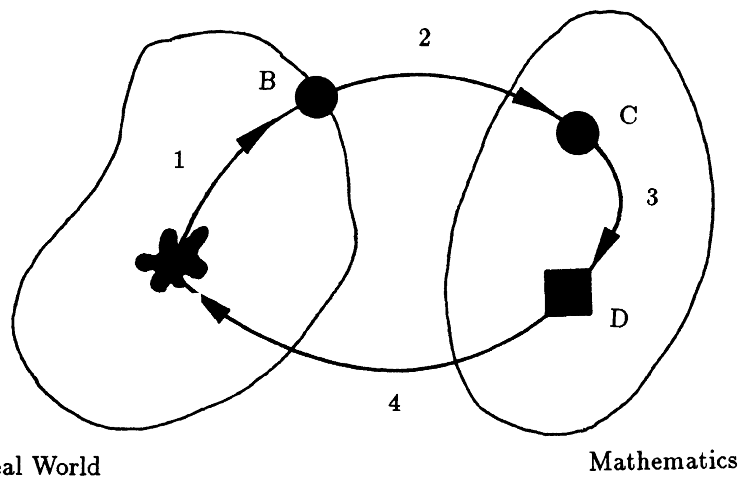
1. Background of the work of the group

1.1 Mathematics and the real world

From its very beginnings, mathematics has been both the most *esoteric* and the most *practical* of human creations. There have been and there are close *relations between mathematics and everyday life, the world around us and other sciences.* Problems in the real world have inspired and stimulated the development of mathematical concepts and theories, and theoretical achievements in mathematics have contributed essentially to solve practical problems.

In the last few decades an enormous *extension* of applicable mathematical topics as well as of disciplines related to mathematics has taken place. Many sciences such as biology, economics and sociology have become more and more *mathematized* (see e.g. Pollak 1979,1988 or Jaffee 1984). This is also and especially due to the rapid development in the field of *computer science.*

As is well-known, there are many simplified models for the complex *interrelations between mathematics and the real world* (for a synopsis, see Kaiser–Messmer 1986). By "real world" we mean the "rest of the world" outside mathematics, i.e. everyday life, the world around us, other disciplines and especially *other school or university subjects.* We choose the following diagram (taken from Blum 1985) as a concise illustration:



A) Real problem situation	1, Specifying, idealizing, structuring
B) Real model	2, Mathematizing
C) Mathematical model	3, Working mathematically
D) Mathematical results	4, Interpreting, validating

The starting point is a "real problem", i.e. a situation in the real world with some open questions. This situation has to be simplified, idealized, structured and made more precise by the "problem solver" according to his/her interests. This leads to a "real model" of the original situation. The real model has to be mathematized, i.e. its data, concepts, relations, conditions and assumptions are to be translated into mathematics. Thus, a "mathematical model" of the original situation results. Then, by working within mathematics, certain mathematical results are obtained. They have to be re-translated into the real world, i.e. to be interpreted in the original situation. In doing so, the problem solver has also to validate the model, i.e. to establish whether he/she can use it for his/her purposes. When validating the model, discrepancies of various kinds can occur which lead to a modification of the model or to its replacement by a new one, i.e. the problem solving process may require going round the loop in the diagram several times. Sometimes, however, even several attempts do not lead to usable result.

The use of this *model conception* of the relationship between mathematics and the real world, especially between mathematics and other subjects, is often very helpful for an adequate solution of a given applied problem. There are, however, also disadvantages. For, by strictly separating mathematics from the rest of the world, inseparable links in content — as they have grown up in many centuries especially between mathematics and physics — are examined in a merely formal manner, i.e., artificial distances between a real situation and its mathematical description are created, e.g. in the case of natural laws.

Besides such complex processes there are also abbreviated and restricted links between mathematics and the real world, especially other subjects: on the one hand a *direct application* of already developed mathematics to real situations with mathematical content, on the other hand a "dressing up" of purely mathematical problems in the words of another subject or of everyday life; such *"word problems"* often give a distorted or falsified picture of reality (which is sometimes done deliberately for didactical purposes).

1.2. Mathematics and other subjects at school and university

At *school and university*, relations between mathematics and the real world, especially between mathematics and other subjects, have mostly played an important role. Here, the lines of development have not taken a straight but rather a "wavelike" course, i.e. there have been phases where extramathematical applications in mathematics instruction or mathematics in the teaching of other subjects were strongly taken into consideration and phases where mathematics was more isolated from other disciplines. In recent years a worldwide trend towards a stronger (re-)emphasizing of applications and links to other subjects as well as an extension of the range of application fields in school and university teaching of mathematics can be observed (cf. Burkhardt 1983, Niss 1987, and Blum/Niss 1989).

When dealing with relations between mathematics and other subjects at school or university, we can distinguish between different aspects (see Niss 1981 and Blum/Niss 1989): Firstly, mathematics instruction may essentially serve two different *purposes*:

(1) to provide learners with knowledge and abilities concerning mathematics as a subject,

(2) to provide learners with knowledge and abilities concerning other subjects to which mathematics is to offer some services.

Secondly, the *organizational framework* of mathematics instruction may take two different shapes:

(a) mathematics may be taught as a separate subject,

(b) mathematics may be taught as a part of and integrated within other subjects.

Thirdly, we have different *educational histories*:

(A) Mathematics in school offering general education, viz. at the primary, lower secondary, and upper secondary level,

(B) mathematics in vocational education,

(C) mathematics in university courses for future mathematicians or mathematics teachers,

(D) mathematics as a service subject in university courses for future scientists, engineers, economists etc.

Now the situation can be illustrated by the following matrix:

	purpose	
organization	(1)	(2)
(a)	examples: (A), (C)	examples: (D), (B), partly (A)
(b)	examples:	examples: (B), partly (D)
	integrated curricula	

In all cells of this matrix, *relations between mathematics and other subjects* may play a role. Also in (a1), examples taken from other subjects may be used for various purposes (see section 3.1). When dealing with (a2), (b1) and (b2), it seems quite natural to include applications from other subjects in mathematics instruction. However, one can sometimes find a "division of labour", both in (a2) and even — on a much smaller scale — in (b2) such that separate mathematics courses devoid of applications are given in order to teach once and for all the mathematical concepts, methods and results needed in the subject being served. In section 3.1 of this report, arguments will be given which will call this approach into question.

Possible relations shown in this matrix also include truly *integrated* curricula (second row), both in school and in university, whereby teaching and learning is taking place in an interdisciplinary and cross-subject way. There are, however, only very few materials and there is even less everyday teaching in this sense.

1.3. The topic of Theme Group T6

Theme Group T6 was dealing with all questions and problems concerning the *relationship between mathematics and other subjects at school and university*, embedded in the more general framework of relating mathematics with the real world as developed in section 1.1. The work of the group concentrated on the *relations* involved, especially on the *role of other subjects for mathematics instruction* and the *role of mathematics for other subjects*, at all levels of the educational system, with particular reference to *mathematics as a service subject*. With respect to the matrix constructed in section 1.2, all four cells were considered, provided that it was possible to distinguish segments of instruction with mathematics as an explicit object of attention. Only such instances of (b1) or (b2) with mathematics totally integrated within other subjects were excluded.

The topics of the group were divided into *two* main *areas*, I and II, one more *"theoretical"* and one more *"practical"*. Area I comprised the following *theoretical and basic aspects*:

I.1. *Historical, epistemological, and methodological aspects* of the relationship between mathematics (instruction) and other subjects.

I.2. *Empirical investigations* into the learning and teaching of mathematics in connection with other subjects.

I.3. The *role of computers* in this field.

I.4. *Recent developments in applications in practice* and their relevance for mathematics instruction.

Area II comprised *examples, materials, and projects* linking mathematics and other subjects, for all educational levels:

II.1. For the *primary* level (5–10).

II.2. For the *junior secondary* level (10–16).

II.3. For the *senior secondary* level (16–19), including vocational education.

II.4. For the *tertiary* level (19^+), with special reference to mathematics as a service subject.

To each of these eight topics, an *organizer* had been assigned beforehand:

I.1: Ubiratan D'Ambrosio (Brazil),
I.2: Gabriele Kaiser-Messmer (FRG),
I.3: Rolf Biehler (FRG),
I.4: Dilip Sinha (India),
II.1: Alan Rogerson (Australia),
II.2: David Burghes (UK),
II.3: Rudolf Bkouche (France),
II.4: Dilip Sinha (India).

1.4. Survey of the work of Theme Group T6

In the *first session* W. Blum gave an introduction to the theme of the group and a survey of the programme. Then U. D'Ambrosio introduced the more theoretical area I of the group (see section 1.3), and A. Rogerson gave a short presentation on this area, dealing with some basic questions in an interdisciplinary curriculum project for schools, the "Mathematics in Society Project" (see Rogerson 1986). Then D. Burghes introduced the more practical area II, and Roger Jean (Canada) gave a short presentation on this area, concerning mathematics as a service subject for biologists (see Jean 1987).

The *second session* was devoted to area I and was divided up into four subgroups according to the points I.1 to I.4 mentioned in section 1.3. After a short introduction given by the respective organizer, in each subgroup there were several short presentations as well as intense discussions. For details see chapter 2 of this report.

The *third session* was devoted to area II and was structured in the same way, according to the points II.1 to II.4. For details see chapter 3.

In the *fourth session* U. D'Ambrosio and D. Burghes summarized the activities of the various subgroups of the second and the third sessions, and this was followed by discussions. Then Jon Ogborn (UK) presented some general reflections on the relations of mathematics and other subjects, especially the sciences, exemplified by the use of computers in data analysis. Finally W. Blum looked back on the topic of the group as well as forwards towards essential activities and research areas for the future. For details see chapter 4.

Some selected papers presented during the sessions of Theme Group T6 will be published in the joint *Proceedings* of Group T6 and Theme Group T3 on "Problem Solving, Modelling and Applications" (Blum/Niss/Huntley 1989).

2. Area I: Theoretical and basic aspects

The following four sections of chapter 2 refer to the subgroups from the second session (see section 1.4). In order to make this report more concrete we will describe one interesting presentation in each subgroup in some more detail.

2.1. Historical, epistemological, and methodological aspects

In his *introductory remarks*, the subgroup organizer, U. D'Ambrosio, emphasized an historical approach to mathematics, recalling that at the outset mathematics was a part of the intellectual efforts of mankind to understand themselves, their environment and their relationship with nature and among one another, as well as to decipher the numerous mysteries posed by nature, by its phenomena and by the universe as a whole. He argued that mankind was driven into understanding and explaining reality, coping with it, managing it, and gaining from it. For this understanding certain techniques were developed, according to diverse cultural contexts. In the course of history, some of these techniques disappeared, some survived or became even stronger, among those being a mode of rational thinking called "mathematics", which includes measuring, counting, classifying, ordering, inferring etc. In summing up, D'Ambrosio stressed that we are therefore also facing an historico-epistemological problem when we discuss the relations of mathematics and other subjects, and interdisciplinarity.

Then three *invited speakers* presented their papers: Günter Ossimitz (Austria) on theoretical mathematical models in economic and management sciences, Jeff Evans (UK) on statistics and the problem of induction, and Shmuel Avital (Israel) on mathematics and cultural values.

As an *example*, we refer here to Ossimitz' contribution. The presenter pointed out that mathematics in economic and management sciences is mostly descriptive mathematics or elementary arithmetic. He emphasized that the fundamental act of mathematization in this field is measurement, through which qualitative stuff is transformed into quantitative structures. In his final thesis he argued that the relation of mathematics to economics is comparable to that of chemistry to medicine.

The *discussion* in this subgroup focussed on the idea of recovering the intimate relations between mathematics and other subjects by recovering the humanistic values of mathematics. Here, the idea of ethnomathematics was also brought forward.

2.2. Empirical investigations

In an *introductory survey* of the state-of-the-art concerning empirical research on the learning and teaching of mathematics in connection with other subjects, given by the organizer G. Kaiser-Messmer, it was pointed out that there are different strands of research, e.g. isolated investigations restricted to quantitative-statistical methods, applied problem solving research, research on curriculum projects based on a theoretical background and closely linked with classroom experiences, and personal reports on certain school or university courses.

The *invited speakers* in this subgroup were Barbara Binns and John Gillespie (UK) on experiences with the "Numeracy Through Problem Solving Project" at the University of Nottingham, Jan de Lange (Netherlands) on a curriculum project on mathematics for the life and social sciences for the upper secondary level, and Christopher Ormell (UK) on research on "application readiness" in mathematics of lower secondary pupils.

As an *example*, we choose J. de Lange's presentation. The author reported on the experiments that eventually led to the introduction of a new curriculum for upper secondary students aiming at a study of the social and life sciences in the Netherlands (cf. de Lange 1987). This curriculum uses the real world as a starting point for extracting mathematical concepts ("conceptual mathematization"). Teachers' and students' reactions were discussed which showed especially a need for many teachers to change their attitudes. In addition, the problem of process oriented assessment was mentioned.

The *discussion* accented three items in particular. Firstly, there is still a considerable gap between everyday school practice and the educational debate on applications to other subjects, but the gap has become reduced during the last few years. Secondly, most students and teachers respond positively to examples taken from other subjects, provided that they are challenging and fit in the syllabus. Thirdly, the ability to link mathematics with other subjects is not at all easy for students and demands special instructional phases.

2.3. The role of computers

For many years now computers have been bursting more and more into many areas of society, including the educational system, and also into mathematics instruction at school and university. The use of computers as a tool, as a means for doing numerical or algebraic calculations or for drawing, as an aid for creating new teaching methods, has implications also for the learning and teaching of mathematics in connection with other subjects (see Blum/Niss 1989).

For example, more complex applied problems with more realistic data become accessible earlier and more easily, or problems which are too demanding can be simulated numerically or graphically. As to goals, routine calculatory skills are becoming more and more devalued and abilities such as modelling, applying or experimenting are becoming revalued upwards. With regard to contents, new topics which are particularly close to applications in other subject areas can be treated more easily now, e.g. data analysis at the upper secondary level or dynamic systems at the tertiary level. Computers entail, however, also many kinds of problems and risks, e.g. the devaluation of routine skills will make mathematics instruction more demanding for all students and too demanding for some of them, for linking mathematics to other subjects is an ambitious activity, with or without computers. And teaching and learning may become even more remote from real life than before, because real life now may only enter the classroom via a computer; simulations may replace real experiments.

In his *introductory remarks* the subgroup organizer, R. Biehler, concentrated on the use of software, especially for modelling and for simulating systems. He reported on promising experiences with using such software in the classroom under certain favourable circumstances, and he also stressed the need for more and deeper empirical investigation in this field. Further, he accentuated the role of computers for deepening the student's understanding of the model conception of the relation between the real world and mathematics.

Four *invited speakers* presented recently developed software and reflections on their possibilities. Jon Ogborn's (UK) lecture focussed on his micro-computer modelling systems DMS and CMS for secondary schools. Two Hungarian Colleagues presented ideas for restructuring the mathematics and the science curriculum in Hungary by using computers for simulation and games, with the emphasis on statistical models. R. Biehler discussed how far computer supported analysis of real data can be helpful for developing an adequate concept of probability.

J. Ogborn *for example* showed that the use of computers gives hope to reducing some technical problems with mathematics, e.g. in postponing analytical methods in favour of discrete methods. Modelling tools may allow a more flexible change and extension of initial models and a numerical and graphical exploration of the consequences of models from different viewpoints. Examples from modelling growth, traffic flow, or atmospheric energy transfer processes were given. So computational modelling serves here as a link between mathematics and science.

In the *discussion*, criteria for appropriate modelling tools were addressed. The need for still using paper and pencil in many situations instead of computers was stressed unanimously. Eventually, the idea of using a "virtual computer" in the classroom was discussed, i.e. to choose approaches to topics and styles in learning and teaching which take into consideration the existence of powerful computers.

2.4 Recent developments in applications in practice

The *introductory talk* by the organizer, D. Sinha, centred essentially around the broad range of recent practical applications of mathematics. He mentioned examples from the physical sciences as well as from engineering, biology, ecology, psychology, communication, and linguistics. Explicitly he spoke about qualitative studies in developmental biology which have led to catastrophe theory and to bifurcation theory, about chaos theory resulting from atmospheric science studies and about fractals in connection with the oceanic sciences. Reference was made to the problem of nonlinearity in physical, social or biological phenomena. An essential aspect of this talk was the role and impact of the new technologies in the practical use of mathematics, e.g. in engineering, medicine or economics.

Furthermore, the speaker reflected on the educational relevance of these newer application areas. He discussed possibilities and strategies for making these areas accessible to learners at the tertiary or even at the secondary level. The idea of joint instruction by a mathematics and a non-mathematics teacher was called to mind and strongly recommended. Sinha regarded such collaborative and interactive ventures as a necessity. Again, he stressed the crucial role of computers, now as a tool for getting examples of recent applications percolated down to the instructional level. He finished his presentation by pleading for an inclusion of such new examples in mathematics curricula in order to cope with the demands of the changing socio-economic and cultural context.

There were three *invited speakers*, R. Jean on recent developments in bio-mathematics and H. Khare together with B. Bawerjee (India) on a survey of some recent examples in applied fields.

In the *discussion*, the participants consented to three recommendations. Firstly, some "leading examples" taken from those newer areas of applied mathematics should be worked out in detail. Secondly, strategies and modalities should be identified to stimulate cooperation and collaboration between mathematics teachers at university and school and their colleagues in other subjects. Thirdly, mathematics teachers should be encouraged to keep uptodate with respect to recent developments in applied fields and to incorporate newer examples into their teaching.

3. Area II: Examples, materials, and projects

The sections 2 to 5 of chapter 3 refer to the subgroups from the third session (see section 1.4). Again, for each subgroup one interesting presentation will be described in more detail. Section 1 deals with some common aspects of this chapter.

3.1. Arguments in favour of applications to other subjects in mathematics instruction

There are many arguments in favour of references to reality in mathematics instruction, especially of connections with other subject areas, mostly agreed among all participants. We briefly refer to five kinds (cf. e.g. Blum 1985 or Niss 1988).

"Pragmatic" arguments: Learners should be taught how to use mathematics to describe special situations taken from other subjects, to understand them better and to cope with them better. This can only be done by dealing with certain applied examples in mathematics instruction.

"Methodological" arguments: Learners should acquire "meta-knowledge" and general capabilities and strategies for applying mathematics. They should learn how to translate between the real world and mathematics, they should reflect on methods of application, and they should come to know possibilities and limitations of the application of mathematics, which includes a critical appreciation of the use or misuse of mathematics. All this can only be achieved by incorporating suitable applied examples into mathematics teaching.

"Formal" arguments: Learners should be taught general "formal" abilities (such as argumentation or problem solving) and attitudes (such as an openness towards problem situations), which can be done also (but not only) by means of examples taken from other subjects.

"Scientific theory" arguments: Learners require a balanced picture of mathematics as a total cultural and social phenomenon, to which *inter alia* references to other subject areas also belong.

"Learning psychology" arguments: Suitable applied examples (as well as suitable purely mathematical examples) can motivate or illustrate mathematical content, can serve in the structuring of larger mathematical subject ranges, can contribute towards better understanding and longer retaining of mathematical topics and can improve the learner's attitude towards mathematics.

These arguments are based on certain *educational aims* which are implicitly contained in the arguments: pragmatic, methodological, or formal aims, aims based on scientific theory or on cultural history. Such aims have different relevance according to different educational histories. From these aims, more *mathematics-oriented aims* can be inferred such as:

- learners should be taught how to handle mathematics in a well-founded and rational manner, particularly with regard to problems in other subjects;
- learners should acquire adequate basic ideas, related to the real world, and basic conceptions with regard to the essential mathematical concepts, methods and results.

There are also some arguments *against* references to reality in mathematics instruction, based on certain obstacles and barriers (see section 4.1). When weighing the arguments and counter-arguments against each other in group discussions, the result was a strong plea for including applications in mathematics instruction.

An important question in many discussions was: where to find *examples* for applications, suitable for teaching? Two very useful resources for materials and literature, relevant to the subject, are the survey articles by Pollak (1979) and by Bell (1983). Further, the extensive bibliography by Kaiser et al. (1982, with a supplement to appear in 1988) should be mentioned. Many references to current curriculum projects as well as to interesting individual contributions can be found in Blum/Niss (1989).

Many more examples for links between mathematics and other subjects were presented in the four subgroups during the third session. The range of application fields comprised non-traditional ones like architecture, art, biology, computing, environment, finance, language, music, and politics. Many of these examples incorporate the use of computers to a substantial degree, e.g. by simulations, spread sheets, or symbolic algebra. We are now going to report on those subgroups.

3.2. The primary level

Invited speakers were Morten Anker (USA) on architectural mathematics, Drora Booth (Australia) on spontaneous pattern painting and Piero del Sedime (Italy) on mathematics and social conditions. The organizer, A. Rogerson, gave an introduction to the topic of the subgroup by stressing the particular advantages of the primary level for linking mathematics with other subjects and especially for integrated curricula, for project work etc.. He also presented the "Mathematics in Society Project" in connection with integrated curricula.

An interesting *example* was M. Anker's presentation. He reported on experiences with children in an "architectural math lab for cubic city planning". Here, primary level pupils designed and built a "children's city" with houses, people, cars etc. in miniature. In doing so, they were inspired to use mathematics creatively as a tool for exploring, describing, and reconstructing their environment. In this project, mathematics was brought together with architecture, art, and social studies.

3.3. The lower secondary level

There were nine *invited speakers*: Emma Castelnuovo (Italy) gave examples of connections between elementary geometry and reality. Ikutaro Morikawa (Japan) showed some real world illustrations of geometric topics. Andrew Begg (New Zealand) discussed possibilities for combining mathematics with Maori language and culture. Maria-Cristina Zambujo (Portugal) gave examples for linking mathematics with biology, ecology, geography, history and languages within a project studying several aspects of a local river, by essentially using computers. Hans-Wolfgang Henn (FRG) presented reflections on and examples for analysing real data. Bruno Vitale (Italy) reflected on the exploration of the space of informatics and the realm of open mathematics. Laurie Aragon (USA) explained some teacher training materials in mathematics applications. David Hobbs (UK) presented the "Enterprising Mathematics" project, a contextual course for the 14–16 year old. Finally Kumiko Adachi (Japan) gave an example for the integration of mathematics with music, design, science and crafts.

From among the various interesting presentations we refer as an *example* to Henn's in more detail. The speaker considered two cases, the measurement of a single value and the investigation of the functional interrelationship of two measured values. His main aim was to show how students should be taught to handle numbers critically in such situations. After specifying his notion of exactness he gave examples of a reasonable calculation of mean values in applied situations and of adequate methods of linearizing given pairs of numbers resulting from measurements of real data, among those a dropping ball or the decay of beer foam.

3.4. The upper secondary level

Seven *invited speakers* gave short presentations: Paul Bungartz (FRG) showed how to use recent real applications in teaching probability. B. Chaudhuri (India) reported on a study on the interaction between languages in the teaching of mathematics and informatics. Solomon Garfunkel (USA) presented the "High School Mathematics and its Applications Project". John Goebel (USA) spoke about an American curriculum project for high schools, with emphasis on applications. Yvette Horain (France) talked about some teaching experiences at the upper secondary level. Bernard Parzysz (France) showed how he teaches solid geometry through shadow problems. Finally Mary Rouncefield (UK) explained a project on the use of statistics in other subjects such as biology, geography, psychology, sociology and economics.

As one of many interesting *examples* we consider Parzysz' talk. His starting point was the "vicious circle" which results from the fact that studying spatial geometry at school requires the drawing of plane projections and vice versa. To break this circle, the author uses shadows cast by an electric light bulb and by the sun. He presented concrete materials and examples used in the classroom, among others cubes, "Dürer's window" and boards.

3.5. The tertiary level

There were seven *invited speakers*: Michel Helfgott (Peru) presented his approach for teaching differential equations to students of science and engineering. Anthony Briginshaw (UK) reported on mathematical language as an information transfer mechanism. Ruth Hubbard (Australia) spoke about incorporating mathematical reading and study skills into mathematics service courses. Megan Clark (New Zealand) analysed factors affecting the flow of students into mathematics, science and technical training. Eric Muller (Canada) raised and discussed some important issues related to service courses in mathematics, starting from experiences in Canada. R. Jean broadened and concretized his presentation given at the first session (see section 1.4). Finally Arno Jaeger (FRG) reported on experiences with a new approach to teaching linear algebra and optimization for beginning students of business administration.

Jean's talk was one of several interesting *examples* for the teaching of mathematics as a service subject, which was the central topic of this subgroup (cf. also Howson et al. 1988 and Clements et al. 1988). Based on aims such as "to instill in students the ability to use the mathematical approach in biological situations", the author pleaded for the so-called "integrated method", where mathematics is taught through biological subject matter in contexts relevant to the undergraduate biology programmes. He gave some examples, taken from genetics or the growth of populations, from the theory of predation or from animal behaviour.

4. Problems and prospects

In many discussions throughout the work of the Theme Group, barriers and obstacles to the linking of mathematics with other subjects were identified. We will briefly refer to some of them in section 1. In section 2 we will enumerate some important activities for the future.

4.1. Obstacles to applications in mathematics teaching

In spite of all the good arguments in favour of applications to other subjects in mathematics teaching (cf. section 3.1), such relations often still do not play as important a role as one would wish in "mainstream" mathematics instruction at school and university. This is due to certain obstacles (well-known amongst mathematics educators for a long time), among others the following (cf. Blum/Niss 1989):

Obstacles from the point of view of instruction: Many mathematics teachers are afraid of not having enough time to deal with applications in addition to the compulsory mathematics material. Some teachers doubt whether relations to other subjects belong to mathematics instruction at all because they would disturb the clarity, the purity, the beauty and the universality of mathematics.

Obstacles from the learner's point of view: Mathematical routine calculations which can be solved by merely following some recipes are more popular with many students than applications, because applications make the mathematics lesson more demanding and less predictable.

Obstacles from the teacher's point of view: Applications also make instruction more demanding for teachers and more difficult to assess. Very often teachers simply do not know enough examples, or they do not have enough time to up-date examples, to adapt them to the actual class and to prepare them in detail.

Participants of the Theme Group agreed that the obstacles related especially to learners and teachers are really serious, but that in the light of the arguments given in section 3.1 mathematics teachers and educators should continue to make every effort to *overcome* these obstacles, especially by an adequate pre-service and in-service teacher education or by stimulating every kind of contact, or rather cooperation, between mathematics teachers at school and university and their colleagues in other subjects. And both teachers and educators should *insist* that *applications* to other subjects *become and remain an essential part* of mathematics instruction, even in the face of what has been mentioned, e.g. that instruction becomes in fact more demanding for students and teachers.

4.2 Future activities

The participants of the Theme Group agreed that the following, among other things, is necessary for all levels of instruction:

1) *To develop more* concrete *"local" examples* for relations of mathematics to other subjects, suitable for teaching. Such examples should be more suited to a motivation or an illustration of certain mathematical topics or to a description and a better understanding of special problem situations taken from other subjects.

2) *To develop more* concrete *"global" examples and project materials* for relations of mathematics to other subjects, suitable for teaching. Such examples should be more suited to developing general abilities such as translating between the real world and mathematics or developing adequate attitudes such as an openness towards problem situations.

3) *To devise more examples and conceptions for the use of computers* in mathematics instruction, with special respect to connections between mathematics and other subjects.

4) *To gain more experiences* regarding both successes and failures in the teaching and learning of mathematics in connection with other subjects, and *to establish* the broadest possible *opportunity to exchange* these experiences.

5) *To carry out more* controlled *empirical investigations* concerning mathematics instruction with respect to relations to other subjects. For instance: what could the actual effects of applications to other subjects be? How do learners react to that? What are the possibilities and risks of the use of computers in an "application-oriented" mathematics instruction?

6) *To connect* more closely *practical teaching experiences* on the one hand and *basic theoretical questions* (such as methodological and epistemological aspects concerning the interaction between mathematics and other subjects, or questions concerning the educational aims of instruction) on the other hand, in both directions.

7) *To embed all reflections and activities* in mathematics education concerning the relation of mathematics to other subjects *into a "theory of mathematics education"*.

All these activities are research activities in a broad sense. But certainly the most important thing to do is still:

8) *To intensify the efforts to integrate applications to other subjects into "standard" everyday mathematics teaching*, by means of curricula, of textbooks and materials for learners and teachers, by pre-service and especially by in-service teacher training.

References

Bell, M. S.: Materials Available Worldwide for Teaching Applications of Mathematics at School Level. *Proceedings of the Fourth International Congress on Mathematical Education* (Ed.: M. Zweng et al.), Boston 1983, 252–267.

Blum, W.: Anwendungsorientierter Mathematikunterricht in der didaktischen Diskussion.*Mathematische Semesterberichte* 32 (1985) 2, 195–232.

Blum, W. et al. (Ed.): *Applications and Modelling in Learning and Teaching Mathematics.* Chichester 1988.

Blum, W.–Niss, M.: Mathematical Problem Solving, Modelling, Applications, and Links to Other Subjects: State, Trends and Issues in Mathematics Instruction. In: Blum–Niss–Huntley (1989).

Blum, W.–Niss, M.–Huntley, I. (Ed.): *Applications, Modelling and Applied Problem Solving: Teaching Mathematics in a Real Context.* Chichester 1989.

Burkhardt, H. (Ed.): *An International Review of Applications in School Mathematics.* Ohio 1983.

Clements, R. R. et al. (Ed.): *Selected Papers on the Teaching of Mathematics as a Service Subject.* Berlin/Heidelberg/New York 1988.

Howson, A. G. et al. (Ed.): *Mathematics as a Service Subject.* Cambridge 1988.

Jaffee, A.: Ordering the Universe: The Role of Mathematics. In: *SIAM Review* 26(1984)4, 473–502.

Jean, R. (Ed.): *Une Approche Mathématique de la Biologie.* Montreal 1987.

Kaiser, G.–Blum, W.–Schober, M.: *Dokumentation ausgewählter Literatur zum anwendungsorientierten Mathematikunterricht.* Karlsruhe 1982 (Addendum 1988).

Kaiser-Messmer, G.: *Anwendungen im Mathematikunterricht (Vol. I/II).* Bad Salzdetfurth 1986.

DeLange, J.: *Mathematics — Insight and Meaning.* Utrecht 1987.

Niss, M.: Goals as a Reflection of the Needs of Society. In: *Studies in Mathematics Education, Vol. 2* (Ed.: R. Morris), Paris 1981.

Niss, M.: Applications and Modelling in the Mathematics Curriculum — State and Trends. *International Journal for Mathematical Education in Science and Technology* 18 (1987) 4, 487–505.

Niss, M.: Aims and Scope of Applications and Modelling in Mathematics Curricula. In: Blum et al. (1988).

Pollak, H. O.: The Interaction between Mathematics and Other School Subjects. In: *New Trends in Mathematics Teaching, Vol. IV* (Ed.: UNESCO), Paris 1979, 232–248.

Pollak, H. O.: Recent Applications of Mathematics and Their Relevance for Teaching. In: Blum et al. (1988).

Rogerson, A.: The Mathematics in Society Project: A New Conception of Mathematics.*International Journal for Mathematical Education in Science and Technology* 17 (1986) 5. 611-616

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Supporting survey presentation:

Werner Blum (FRG) and *Mogens Niss* (Denmark) presented a joint survey lecture for Theme Groups T3 and T6. Selected papers of the speakers in these groups are planned to be published in a separate volume.