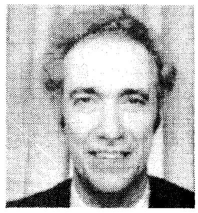
In: Teaching mathematics and its applications, Editor: Burghes, Volume 10, Number 1, 1991, OXFORD UNIVERSITY PRESS, S. 112-123

Teaching and Learning of Mathematics and its Applications: First Results from a Comparative Empirical Study in England and Germany



WERNER BLUM, DAVID BURGHES, NIGEL GREEN and GABRIELE KAISER-MESSMER

1. Background

THE philosophy behind the educational system in England and Wales is currently undergoing fundamental change. A national curriculum has been introduced, and methods of assessment at four key stages

KS1: age 7, KS2: age 11,
KS3: age 14, KS4: age 16

are being developed which look likely to be based principally on external nationwide tests. Part of the argument used by the government for radical change of the system is based on compatibility with other countries—in particular, Germany and Japan are regarded as having more effective systems of education which provide a more capable and efficient workforce for the future.

This is the background to a *comparative project* in which we are seeking to make comparisons between the German and English approaches to teaching mathematics and its applications. Although comparisons have been used extensively in many recent documents in the UK (see, for example, Prais and Wagner[1]), the evidence has often been at best based on assumptions that are readily criticized, and often no more than anecdotal. Our aim in this project is to provide more substantive evidence on the effectiveness of different teaching approaches to mathematics, and particularly in relation to teaching mathematics through its applications and uses. The project is based on collaboration between the University of Exeter in England and the University of Kassel in Germany.

Mathematics teaching worldwide has undergone some fairly major changes in the past 50 years. The 1960s led to the promotion of so called "modern" mathematics, but the international environment in the last 15 years has emphasised the "applicability" of the subject (e.g., among others, Burghes[2]; for a detailed survey see Blum and Niss[3]). There have been two international studies in mathematics which have made comparisons between a number of countries[4], but there was little emphasis put on the applications of mathematics. In the UK, the Assessment of Performances Unit regularly monitors "standards" in a number of broad areas, but again applying and using mathematics in context is not one of them[5]. In Germany there have been few empirical investigations (see, e.g., Kaiser-Messmer[6]) and, up to now, no comparative empirical research on the effectiveness of different approaches to teaching applications of mathematics has been carried out.

The project has a number of phases of which only the first has currently been completed. In this article, we will outline the results from the first phase, in which

(i) comparison between the different educational systems are made;
(ii) hypotheses about the effectiveness of the German and English approaches to teaching applications of mathematics are postulated; and
(iii) empirical data based on case studies are analysed.

We will also describe the ongoing research taking place in the next phases of the project.

2. Educational framework

Any comparative empirical study in German and English classrooms must take into account the differences between the two school systems. In England, the great majority of secondary children (age 11–18) are in "comprehensive" schools, which cater for a full range of ability. In many areas, secondary schooling consists of a comprehensive school up to the age of 16, followed by a tertiary/sixth form college for the next two years. A small but significant minority of pupils attend independent schools and a number of new types of schools are currently being developed (e.g., City Technology Colleges, grant-maintained schools).

In Germany, there are mostly three types of secondary schools, namely,

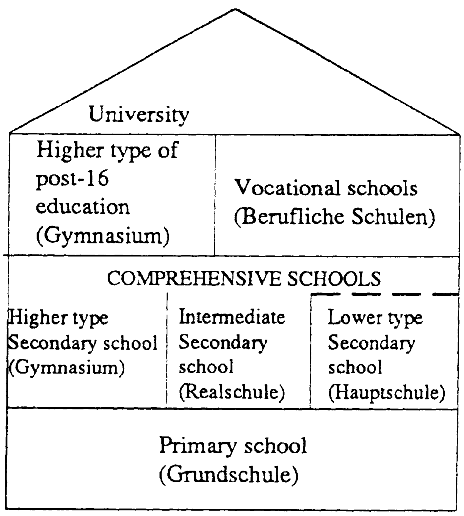
Gymnasium (age 11–19)	Comparable with Grammar Schools in England, leading to the "Abitur" (University entrance examination)
Realschule (age 11–16)	Schools for "average" pupils, partly academically orientated, partly vocationally based
Hauptschule (age 11–15/16)	Schools for the least able

In some parts of Germany, comprehensive schools exist, too. In one type, these three systems are combined within one school, but remain separated—"Additive Gesamtschule." Here pupils are normally taught as a class for all subjects. In another type, pupils are mostly instructed jointly and only some main subjects (especially mathematics) are taught at different ability levels—"Integrierte Gesamtschule."

It should also be noted that a year group in Germany often has a number of older pupils in it. This occurs when pupils fail to gain sufficiently high grades during the year: they must then repeat the year ("sitzenbleiben"). Figure 1 illustrates the educational system in Germany.

Furthermore, the German school system is characterised by the compulsory nature of the curriculum and compulsory core subjects, whereas in the English school system up until now the curriculum is flexible, mostly developed by the schools, and many subjects are optional. Therefore, an early specialisation takes place in English schools, unlike in the German system, which emphasises a broad general education (for details see Hearnden[7] and Glowka[8]).

It is obvious that any comparisons that are made must be put into the context of vastly different educational frameworks. McLean has recently made a study of the German and English educational philosophies. He describes the English tradition of education as influenced by three major principles, namely, (i) morality (ideal of the Christian gentleman), (ii) individualism, and (iii) specialisation. He argues that these principles have given little weight to rational, methodical and systematic knowledge. The introduction of the national curriculum, though, should give more emphasis to these points. He further argues that mathematics teaching in English schools has little concern for the formal learning of principles and insight into general mathematical rules, but is based



Year	Age	School level
13 12 11	18 17 16	Upper secondary level
10 9 8 7 6 5	15 14 13 12 11 10	Lower secondary level
4 3 2 1	9 8 7 6	Primary level

Fig. 1

on the assumption that children grasp general principles as a result of active discovery and active work through a series of examples. This has led to the wide use of example-based work as well as individualised work in English mathematics instruction. The idea that mathematics can promote general goals such as developing logical thinking has been largely ignored, whilst the use of mathematics in later life has been emphasised.

On the other hand, McLean characterises the German educational philosophy as encyclopaedic, with connections to naturalism. The German educational philosophy is dominated by the principles of universality and rationality, which has led to a high relevance for general education since Humboldt (originally only available for the elite). For mathematics teaching, these aims place the understanding of structures and general principles in the foreground and lead to a low importance being attached to active work through examples. In the *holistic* view of knowledge, the understanding of structures is seen to be more important than deep knowledge in single areas.

The naturalistic view aims to connect school life with the community and environment. It is with middle- and low-ability students that this view begins to outweigh the rational approach: here work-orientated schemes have been developed, which emphasise the connection of school knowledge with everyday life and work (e.g., by Kerschensteiner).

3. Approaches to teaching mathematics and its applications

The varying educational framework and philosophies in England and Germany are apparent in the way that mathematics and its applications are taught in the schools involved in the study. In addition to the methodological aspects mentioned above (e.g., the role of active individual work of students), different approaches to teaching mathematics can also be characterised by underlying aims of mathematics teaching. Blum[10] distinguishes between three general aims:

- *Pragmatic* aims: Mathematics as an aid for describing, understanding, and mastering real world problems and situations.
- *Formal* aims: Mathematics as an aid for developing general skills, e.g., the ability to translate between the real world and mathematics, or attitudes such as willingness for intellectual effort.
- *Cultural* aims: Mathematics as a source for reflection, and mathematics as a science and a part of human history and culture.

The German approach to teaching mathematics and its applications, especially at the upper ability level (Gymnasium), normally stresses cultural and formative aims. It has strong connections to the so-called *scientific-humanistic* trend in the international mathematics education debate. The English approach puts more emphasis upon pragmatic aims and therefore adheres, on the whole, to the so-called *pragmatic* trend although there has been recent emphasis on formal goals as well (for details, see Kaiser-Messmer[11]).

These two approaches led us to ask questions such as the following.

1. Is the English approach more appropriate for promoting the ability to apply mathematics to real-world problems?
2. Does the English approach build up barriers for a comprehensive understanding of concepts in mathematics?
3. Is the German approach more appropriate for developing a balanced image of mathematics as a science?
4. Does the German approach provide an inadequate background for tackling real-world problems?

These were some of the questions that we sought to gain insight into in this study.

4. Methodology

The first phase of our project has mostly been based on a case study approach, because quantitative-statistical methods are hardly possible on the basis of the present research knowledge. The case study method is thought to be especially suitable for generating research-guiding hypotheses which match the exploratory character of the project. In the last few years it has been shown that the case study method is suitable for comparative empirical research (see, for example, Stenhouse[12] and Crossley and Vulliamy[13]).

So far in this research, several case studies in English and German school classes (pupils aged 14–16) have been carried out based on teaching materials which are characteristic either of the pragmatic or the scientific-humanistic method. The case studies are limited to the Higher stream of (English or German) comprehensive schools or the (German) Higher school type at Lower secondary level (Gymnasium).

As a research method, the techniques of classroom observations, attitude tests, and achievement tests have been used. The attitude tests have been used to find out about the image of mathematics and the interest in and attitude towards mathematics, and the achievement tests have been used to assess problem solving abilities and concept comprehension. An example of one of the attitude and achievement tests is given in the Appendix.

The empirical investigations in English classrooms were based on an experimental teaching course developed at the Centre for Innovation in Mathematics Teaching ("Enterprising Mathematics Course"), covering year groups 10 and 11 before the General Certificate of Secondary Education (GCSE). This course is typical of recent English experimental approaches to the teaching of mathematics in context, because of its strong emphasis on the development of mathematical concepts out of real-world contexts (for details, see Hobbs and Burghes[14]). In Germany, mathematics teaching is in most cases strongly based on textbooks (which have to be approved by the local "Bundesland" and have to match the local mathematics curriculum). There is a great variety of textbooks in Germany. We observed classes using the texts "Mathematik heute." In addition, some classes used materials from "Mathematikunterrichtseinheitendatei." The teaching examples we observed are typical for the German situation; that is, to develop mathematical concepts within real-world contexts with an emphasis on different basic mathematical ideas and to consider a broad range of goals. Furthermore, the examples are characterised by their relations to the environment (for examples, see Kaiser-Messmer *et al.*[15]).

In this first phase, two comprehensive comparative case studies have been carried out (with 61 German and 94 English students). In the first study, two groups, year 10, of the Higher stream of an English comprehensive school were observed for four weeks, during which the surface and volume of geometrical solids was dealt with mathematically. In a parallel study with German students, three groups, year 10, from the Higher type secondary school were observed dealing with the same mathematical topics. The English and the German classes used similar types of real-world examples out of the themes "packaging" and "buildings." The English students spent a considerable number of their teaching lessons occupied with coursework on the theme "packaging." In the second case study, two English groups, year 11, from the Higher stream of an English comprehensive school and one group, year 10, from the Higher stream of a German comprehensive school were observed (again for four weeks). The course dealt with trigonometric functions in which similar types of real-world examples were discussed with trigonometric functions in which similar types of real-world examples were discussed from physics and everyday life (including oscillations of tuning forks and the daily temperature variation). In the German group, examples from everyday life were given more emphasis, whereas in the English groups more physical examples were dealt with. Additionally, in a smaller comparative study, the teaching work with the same (translated) teaching units (Bode's law, and testing physical fitness) were observed in English groups (year 10/11) as well as in German groups (year 9/10) (the original English versions are given in the Appendix).

5. Results from the first phase of the project

The first preliminary results from the classroom observations can be summarised as follows.

A. Educational framework

(i) The greater curricular freedom in the English school system, and in particular in mathematics teaching, than in the German school system is restricted in years 10 and 11 by the General Certificate of Secondary Education, which as an external examination, exerts pressure on the school and the teachers as well as influencing the teaching style and the teaching content considerably.

(ii) In the German school system the regular testing and the half-yearly reports put a continuous pressure on the working attitude of the students.

B. Teaching styles

(i) In English mathematics teaching, class discussion between the teacher and the students as a group takes place considerably less often than in Germany. This is not only characteristic of mathematics teaching but is characteristic of the typical English teaching style, in which an explanation of the problem at the beginning is given, followed by individual work, helped by the teacher, when necessary. Difficulties with the solution of the problem are sometimes not discussed with the whole group, but *individually* between teacher and student. In German mathematics teaching great store is set by the class discussion, which means that explanation of the problems and introduction of new methods usually take place in a teacher-led discussion with the whole group. Consequently the quality of the lessons in German schools rests heavily on the quality of the class discussion and thereby on the ability of the teacher to organise it.

(ii) Mathematics teachers in German schools dictate the interaction and pace in class more than mathematics teachers in English schools. This leads to distinct differences, so that in German classes many mathematical topics are often treated in a shorter time than in English classes.

In English classes observed, teachers often had difficulties in adequately directing the individualised work of all the students. The teachers are not always able to structure the work of each student, as they are on different routes and levels, so that many students are dependent on just the teaching materials (and their quality).

(iii) It is common in English schools to collect the exercise books of the students and check both their classwork and homework. This does not happen so often in German schools, but German mathematics teachers control homework more strictly within the lesson and take action when homework is not done.

C. Teaching content and use of real-world examples

(i) In mathematics teaching in German schools, short problems with definite solutions dominate against comprehensive problems. In contrast, comprehensive, open-ended problems with more than one solution are very often considered in mathematics lessons and associated coursework in English schools. Owing to the crowded curriculum and lack of time, the German mathematics lessons rarely deal with projects or modelling examples, (although many ideas have been published to date, even in textbooks).

(ii) English mathematics lessons more often deal with real-world examples than German mathematics lessons. Examples are different in type and style from German examples; they are very often practical and investigational, whereas in German lessons structured, less-practical examples dominate. On the other hand, real-world examples serve different purposes: in English lessons real-world examples are important for cross-curricular goals; for example, for promoting the general development of strategies and problem solving methods. In German mathematics lessons, real-world examples mainly serve mathematical purposes such as reinforcement of a mathematical concept.

(iii) In German mathematics lessons great store is set by precise and correct mathematical speech and writing, whereas in English mathematics lessons it is often regarded as "old-fashioned" to pay particular attention to precise and correct formulation.

(iv) The German mathematics teaching is characterised besides its conceptual precision, by its orientation towards mathematical structure. On the whole it is common for a large area of mathematics to be taught in "one go" in Germany, whereas in England it is usual for the subject matter (for example trigonometry) to be delivered through the years in small "parcels," which can destroy the overall understanding of the topic. The different orientation between English and German mathematics teaching is readily recognisable by the headings of the materials used. In the English mathematics lessons the underlying mathematical structure is very often ignored, with many small pieces of theory being taught but not being connected together, either in the teaching or in the teaching materials.

D. Attitude interviews and achievement tests

The first preliminary results from the attitude interviews and the achievement tests covering mathematical and extramathematical topics show that there are significant differences between the German and the English students but, perhaps unexpectedly, many common points. A summary of some of these results is given below, whilst a more detailed description will be given in a separate article (to appear). The English versions are given in the Appendices.

(i) *Attitudes towards mathematics.* Very few differences between the German and English students were found concerning attitudes towards mathematics. More than half of the students, the German as well as the English, expressed interest in mathematics. More English students emphasised as the reason for their interest in mathematics that they like problem solving and investigations, whereas only German students named the logical structure of mathematics as a reason.

About two-thirds of the English as well as the German students saw a big difference between mathematics and other subjects. But in the differences named, only German students emphasised aspects of mathematics as a science such as the logical structure or the theory-orientation, whereas more English students named aspects concerning the teaching–learning process. About two-thirds of the English as well as the German students described mathematics as highly important for life, although most of the students were not able to give examples in which they have used mathematics in daily life or other subjects. They only were able to name areas like shopping or banking.

(ii) *Achievements in mathematics and extra-mathematical fields.* A quite surprising result of the test has been that a few weeks after finishing the teaching unit the German as well as the English students were only able to solve elementary mathematical and real-world problems from the unit. Complex problems which had been covered in the regular tests (usual in the German school system) or in coursework could only be recalled by the most able students. Nevertheless, many of the achievement differences which were observed between the English and the German students could readily be explained by the teaching approach, which had or had not emphasised particular points or trained for special tasks. On the whole though, especially with the more complex tasks, the German students obtained better results.

The German students showed a greater familiarity with rearranging formulae, whereas most of the English students, even the weaker ones, were able to solve some of the problems with the help of examples. The same observation could be made with the understanding of formulae: the most able German students were able to explain how special formulae (such as the surface area of a cone) had been derived in the mathematics lessons. On the other hand, more English than German students were able to remember those formulae, which have been derived in a more practical way in English lessons. The more precise, but more abstract explanations, more common in German mathematics lessons, seemed to be more suitable for the most able students than for the majority.

The achievement differences in applied problem solving were significantly smaller than expected, especially with open-ended problems, in which both groups of students had severe difficulties. Only a few of the very able students were able to cope with such problems; this observation points to a strong relation between general mathematical and applied problem solving abilities.

Finally, it was noted that there were significant differences in the approaches to working through these tests: German students worked from the beginning through the test and broke off at some point, whereas English students "jumped" from problem to problem trying to find solutions for any of the tasks, and not being as easily frustrated as the German students when they failed to solve the whole problem.

6. The next phase of the project

Whilst the study so far has proved interesting, and we have been able to make some positive statements, we have also concluded that a more comprehensive survey needs to be undertaken in order to measure and monitor student progress and to seek the relationship of this with the various factors that can influence mathematical development.

Consequently, we are now extending the original aims of the project, and proposing to follow for a period of several years cohorts of students in a number of contrasting schools in both England and Germany. Each year their mathematical progress will be monitored, and progress measured against various school and home factors. In this way, we hope that we will be in a position to be more precise in describing the effect of both common features and differences in English and German mathematics teaching and learning, and also in making recommendation for good practice in teaching mathematics and its applications.

References

1. Prais, S. and Wagner, K., *National Institute Economic Review*, May 1985, 53–76.
2. Burghes, D., *Educational Studies in Mathematics*, 1980, **11**, 113–131.
3. Blum, W. and Niss, M., *Educational Studies in Mathematics*, 1991, **22**, 37–68.
4. Cresswell, M. and Gubb, J., "The Second International Mathematics Study in England and Wales," NFER-Nelson, Windsor, 1987.
5. Foxman, "A Review of Monitoring in Mathematics," Assessment Performance Unit, 1985.
6. Kaiser-Messmer, G., Application-orientated mathematics teaching. In "Applications and Modelling in Learning and Teaching Mathematics," Blum, W. *et al.*, Editors, Ellis Horwood, Chichester, 1989, pp. 66–72.
7. Hearnden, A., *Oxford Review of Education*, 1986, **12**, 187–194.
8. Glowka, D., *Comparative Education*, 1989, **25**, 319–332.
9. McLean, M., "Britain and a Single Market Europe, Prospects for a Common School Curriculum," Kogan Page, London, 1990.
10. Blum, W., Applications and modelling in mathematics teaching—A review of arguments and instructional aspects. In "Teaching of Mathematical Modelling and Applications," Niss, M., Blum, W. and Huntley, I., Editors, Ellis Horwood, Chichester, 1991, pp. 10–29.
11. Kaiser-Messmer, G., Application-orientated mathematics teaching. A survey of the theoretical debate. In "Teaching of Mathematical Modelling and Applications," Niss, M., Blum, W. and Huntley, I., Editors, Ellis Horwood, Chichester, 1991, pp. 83–92.
12. Stenhouse, L., *Comparative Education*, 1979, **15**, 5–10.
13. Crossley, M. and Vulliamy, G., *Comparative Education*, 1984, **20**, 193–207.
14. Hobbs, D. and Burghes, D., Enterprising mathematics: A cross-curricular modular course for 14–16 year olds. In: "Applications and Modelling in Learning and Teaching Mathematics," Blum, W. *et al.*, Editors, Ellis Horwood, Chichester, 1989, pp. 159–165.
15. Kaiser-Messmer, G., Blum, W. and Schober, M., "Dokumentation ausgewahlter Literatur zum anwendungsorientierten Mathematikunterricht," Vol. 2. Fachinformationszentrum Energie Physik Mathematik, Karlsruhe, 1990.

David Burghes is currently professor of education at Exeter University, with responsibility for mathematics, science and technology. He is also director of the Centre for Innovation in Mathematics teaching, and his main crusade is to make mathematics more enjoyable and accessible for everyone. Before moving to Exeter he spent five years working at Cranfield, which strongly influenced his views on mathematics and education. He has published books on mathematics for both schools and higher education, loves travelling, and reads the British Rail timetables for relaxation.

Gabriele Kaiser-Messmer graduated from the University of Kassel with an MEd in Mathematics and Humanities. After teaching in schools for a number of years, she has spent the past ten years working on research projects in mathematics education. She completed her PhD in 1986 and recently has been working on comparisons of different international approaches to the teaching of mathematics. She has published many articles and given talks at a number of recent international conferences.

Nigel Green graduated from Nottingham University and has taught in comprehensive schools in Cheshire and Bradford; he is currently Head of Maths at Evesham High School. His wife is German and they have two children; he spent three years working in finance in Germany. He is a member of the Spode Group and is particularly interested in applications of mathematics.

Werner Blum has been Professor of Mathematics and Mathematics Education at Kassel University (Germany) since 1975. He was a Visiting Professor at Dortmund University in 1983/84, at Linz University (Austria) in 1985 and 1989, and several times at King Mongkut's Institute of Technology, Bangkok (Thailand). He received a Diploma (MSc) in Mathematics 1969 and a Dr. rer. nat. (PhD) also in Mathematics, both from Karlsruhe University. He has written and edited several books and numerous articles and research papers on mathematics education. The focal points of his research and development work are applications and modelling in mathematics teaching, mathematics in vocational education, and the teaching of calculus. He has been an enthusiastic football player since he was three years old, and also likes playing tennis and table tennis.

Address for correspondence: David Burghes, University of Exeter, St. Luke's, Exeter EX1 2LU, UK.

Appendices

TESTING PHYSICAL FITNESS 1/8

P.E. experts have spent much time trying to find a single test which best measures physical fitness.

Research has shown the connection between inactivity and coronary heart disease, and so emphasis has been placed on taking regular exercise. A key concept in testing physical fitness is that of a person's *pulse rate*, and in particular how quickly it returns to its normal rate after excessive exercise.

It is vital that the pulse rate returns quickly to normal after strenuous exercise, otherwise the heart is put under continuous stress.

GALLAGHER AND BRAIHE TEST

In this test, the person exercises on a bench (or stair) 18 inches for boys, 16 inches for girls - the tester shouts out "up - 2 - 3 - 4 " continuously. The "up" command coming every 2 seconds for 4 minutes, the person continuing for as long as possible up to the complete four minutes.

The **pulse rate** is taken at the following times after the person stops exercising:

1 - 1½ minutes; 2 - 2½ minutes; 3 - 3½ minutes.

In each case the number of beats in the half minute is multiplied by 2 to give the pulse rate.

The **fitness index** is evalulated from

$$\text{Index} = \frac{50 \times T}{(p_1 + p_2 + p_3)}$$

where T is the duration of the exercise in *seconds*, and p_1, p_2, p_3 are the measured pulse rates. The grading is given in the table below

Index	Grade
< 50	Very poor
50 - 60	Poor
60 - 70	Fair
70 - 80	Good
80 - 90	Excellent
> 90	Superb

Use this index to test your physical fitness, and keep testing over a period of time to monitor your progress.

EXTENSION

Construct a graph of T against total pulse rate to show the different grades.

QUESTIONNAIRE ON MATHEMATICS

TOPIC UNIT Containers

Name ____________________

School ____________________

Year __________ **Set** __________ **Male/Female** __________

Exam Board ____________________

Please answer the following questions, ticking the appropriate boxes where necessary.

SECTION A Mathematics in General

A1 Are you interested in Mathematics? ☐ **No** ☐ **Don't mind** ☐

A2 Are mathematics lessons different from other school subjects? **Yes** ☐ **No** ☐

A3 Do you use Mathematics in everyday life? Yes ☐ No ☐ If yes, please give some examples below.

A4 Do you find Mathematics helpful in other subjects in school? Yes ☐ No ☐
If yes, please list those subjects below.

A5 Do you find that problems are easier ☐ more difficult ☐ to solve if they are concerned with real life situations?

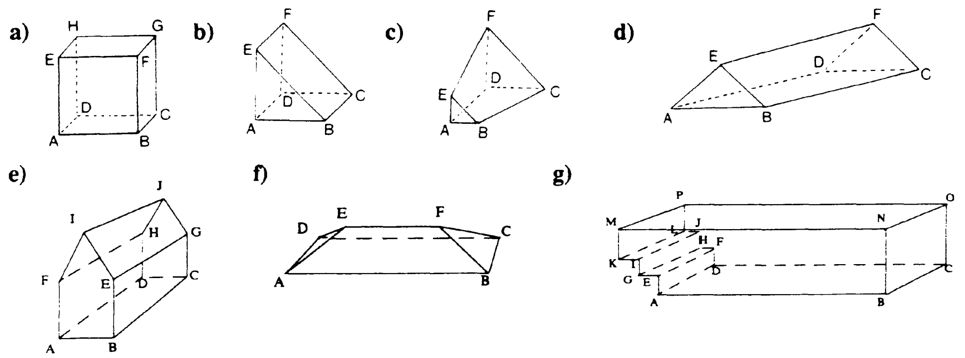
SECTION B : Topic Unit - Containers

Note * means a difficult problem.

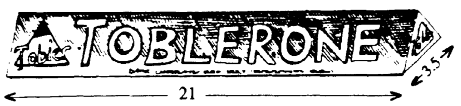
Pyramid	Volume = $^1/_3$ x base area x perpendicular height
Prism	Volume = area of cross-section x length
Cylinder	Volume = area of circular end x height = $\pi r^2 h$ Curved surface area = circumference of circular end x height = πdh
Cone	Volume = $^1/_3$ x base area x perpendicular height = $\pi r^2 h/3$ Curved surface area = π x base radius x slant height = πrl
Sphere	Volume = $^4/_3 \pi r^3$ Surface area = $4\pi r^2$
Triangle	Area = $^1/_2$ x base x perpendicular height
Trapezium	Area = $^1/_2$ x sum of parallel sides x distance between them
Circle	Circumference = π x diameter = πd Area = π x (radius)^2 = πr^2

Where necessary, use the value for π given by a calculator and round answers to a reasonable degree of accuracy.

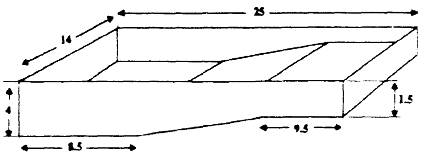
B1 Which of these solid shapes are prisms? Mark in the picture **one** cross-section.



B2 Find the volume of the Toblerone chocolate packet and the area of material needed to make it. (measurements in cm)



* **B3** How many litres of water are needed to fill the swimming pool?
(measurements in metres)



B4 a) A lemonade drink is sold in a cylindrical 0.33 litre can and also in a 0.5 litre can.
Both cans have the same diameter of 6.4 cm.
Find the **height** of both cans.

b) In aeroplanes the same lemonade is offered in a smaller 150 millilitre can with a height of 7.6 cm. Find the diameter of this can.

c) Rearrange the formula for the volume, V, of a cylinder to find
(i) the height h when the volume V and the radius r are given.
(ii) the radius r when the height h and the volume V are given.

B5 How does the volume of a cylinder change, when

a)	the height is	(i) doubled	(ii) trebled
b)	the radius is	(i) doubled	(ii) trebled
c)	the height and the radius are both	(i) doubled	(ii) trebled?

Give a general rule to describe the changes.

B6 The volume of a pyramid equals $^1/_3$ x base area x perpendicular height.
Give reasons for this rule. (Try to remember the explanations which have been given in the topic unit.)

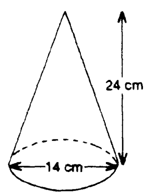
B7 The roofs of the three towers of the cathedral in Limburg are to be re-slated.

The measurements of the roof of the big tower in the middle are:
edge length of base = 415 m
perpendicular height = 12m.

The measurements of the roofs of the two smaller towers are:
diameter = 8 m
perpendicular height = 10 m

How many m^2 of slates will be needed altogether?

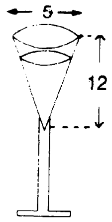
B8 Sketch a net of this cone.
Include any necessary measurements.



* **B9** The curved surface area of a cone equals $\pi \times$ base radius x slant height.
Try to prove it.

* **B10** a) A sparkling wine glass is filled to half of its height. To what percentage of its volume is the glass filled?

b) Find the height of the sparkling wine when the glass is a quarter full.



* **B11** The lung of a human being contains approximately 400 000 000 small bubbles. One bubble has a diameter of 0.2 mm.

a) Find the surface area of all these bubbles. What assumptions do you make?

b) Find the diameter of a single sphere with the same surface area.

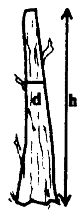
c) Find the surface area of a single sphere having the volume of all the lung bubbles together.

* **B12** In forestry, a rule of thumb is used to determine the volume of a tree to be cut. This is based on the diameter in the middle of the tree and the height of the tree.

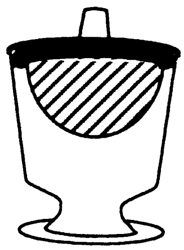
a) Develop such a rule of thumb.

b) Why is this rule of thumb not accurate?

c) How would it be possible to find the volume more precisely?



* **B13** The picture shows a cross-section of a jar of face cream. Only the hatched part is filled with cream.



Describe how you might find, approximately, the volume of the jar which does **not** contain cream.